WORK

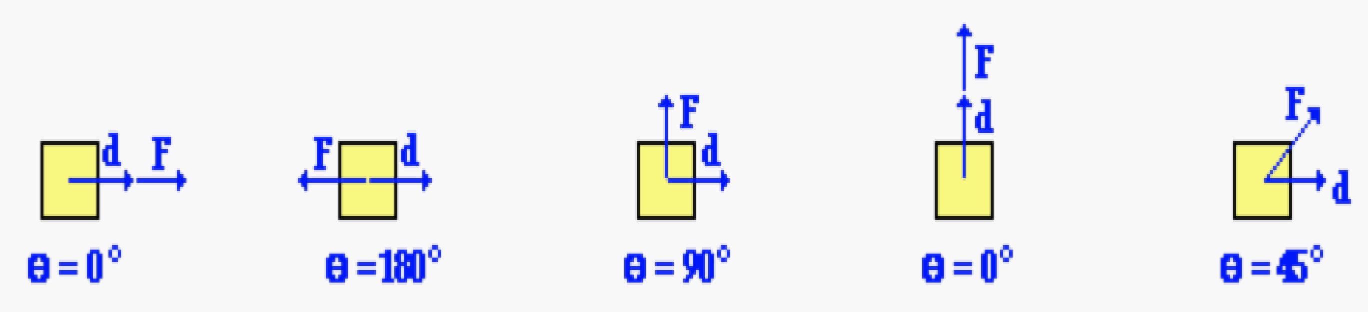
Work results when a force acts upon an object to cause a displacement (or a motion) or, in some instances, to hinder a motion. Three variables are of importance in this definition - force, displacement, and the extent to which the force causes or hinders the displacement. Each of these three variables find their way into the equation for work. That equation is:

Work = Force • Displacement • Cosine(theta)

W = F • d • cos(theta)

Since the standard metric unit of force is the Newton and the standard meteric unit of displacement is the meter, then the standard metric unit of work is a Newton•meter, defined as a Joule and abbreviated with a J.

The most complicated part of the work equation and work calculations is the meaning of the angle theta in the above equation. The angle is not just any stated angle in the problem; it is the angle between the F and the d vectors. In solving work problems, one must always be aware of this definition - theta is the angle between the force and the displacement which it causes. If the force is in the same direction as the displacement, then the angle is 0 degrees. If the force is in the opposite direction as the displacement, then the angle is 180 degrees. If the force is up and the displacement is to the right, then the angle is 90 degrees. This is summarized in the graphic below.



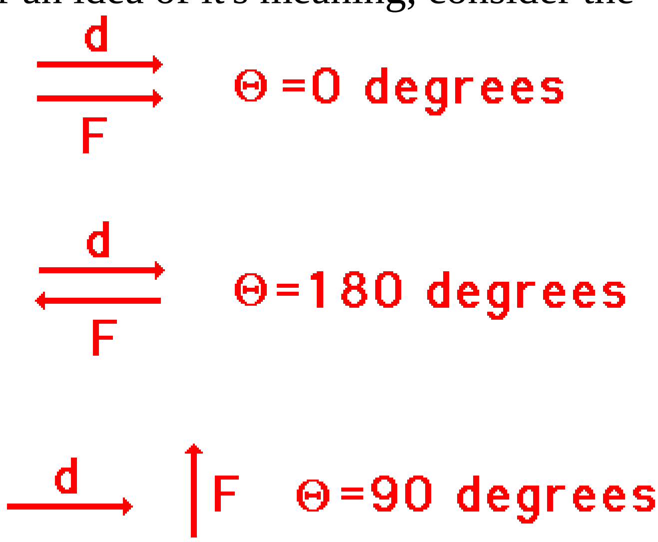
When a force acts upon an object to cause a displacement of the object, it is said that work was done upon the object. There are three key *ingredients* to work - force, displacement, and cause. In order for a force to qualify as having done *work* on an object, there must be a displacement and the force must *cause* the displacement. There are several good examples of work that can be observed in everyday life - a horse pulling a plow through the field, a father pushing a grocery cart down the aisle of a grocery store, a freshman lifting a backpack full of books upon her shoulder, a weightlifter lifting a barbell above his head, an Olympian launching the shot-put, etc. In each case described here there is a force exerted upon an object to cause that object to be displaced.

Work Equation

Mathematically, work can be expressed by the following equation.

$W = F \cdot d \cdot \cos \Theta$

where F is the force, d is the displacement, and the angle (theta) is defined as the angle between the force and the displacement vector. Perhaps the most difficult aspect of the above equation is the angle "theta." The angle is not just *any 'ole angle*, but rather a very specific angle. The angle measure is defined as the angle between the force and the displacement. To gather an idea of it's meaning, consider the following three scenarios.

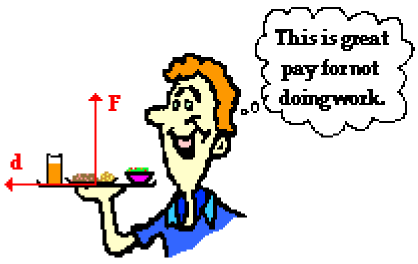


- Scenario A: A force acts rightward upon an object as it is displaced rightward. In such an instance, the force vector and the displacement vector are in the same direction. Thus, the angle between F and d is 0 degrees.

- Scenario B: A force acts leftward upon an object that is displaced rightward. In such an instance, the force vector and the displacement vector are in the opposite direction. Thus, the angle between F and d is 180 degrees.

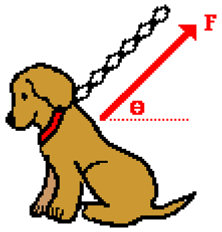
- Scenario C: A force acts upward on an object as it is displaced rightward. In such an instance, the force vector and the displacement vector are at right angles to each other. Thus, the angle between F and d is 90 degrees.

To Do Work, Forces Must *Cause* Displacements



Let's consider Scenario C above in more detail. Scenario C involves a situation similar to the waiter who carried a tray full of meals above his head by one arm straight across the room at constant speed. It was mentioned earlier that the waiter does not do work upon the tray as he carries it across the room. The force supplied by the waiter on the tray is an upward force and the displacement of the tray is a horizontal displacement. As such, the angle between the force and the displacement is 90 degrees. If the work done by the waiter on the tray were to be calculated, then the results would be 0. Regardless of the magnitude of the force and displacement, F*d*cosine 90 degrees is 0 (since the cosine of 90 degrees is 0). A vertical force can never cause a horizontal displacement; thus, a vertical force does not do work on a horizontally displaced object!!

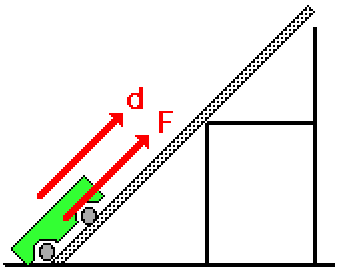
It can be accurately noted that the waiter's hand did push forward on the tray for a brief period of time to accelerate it from rest to a final walking speed. But once *up to speed*, the tray will stay in its straight-line motion at a constant speed without a forward force. And if the only force exerted upon the tray during the constant speed stage of its motion is upward, then no work is done upon the tray. Again, a vertical force does not do work on a horizontally displaced object.



Only the horizontal component of the force (F•cos θ) causes a horizontal displacement.

The equation for work lists three variables - each variable is associated with one of the three key words mentioned in the definition of work (force, displacement, and cause). The angle theta in the equation is associated with the amount of force that causes a displacement. As mentioned in a previous unit, when a force is exerted on an object at an angle to the horizontal, only a part of the force contributes to (or causes) a horizontal displacement. Let's consider the force of a chain pulling upwards and rightwards upon Fido in order to drag Fido to the right. It is only the horizontal component of the tension force in the chain that causes Fido to be displaced to the right. The horizontal component is found by multiplying the force F by the cosine of the angle between F and d. In this sense, the cosine theta in the work equation relates to the *cause* factor - it *selects* the portion of the force that actually causes a displacement.

The Meaning of Theta



Whenever F and d are in the same direction, θ = 0 degrees.

When determining the measure of the angle in the work equation, it is important to recognize that the angle has a precise definition - it is the angle between the force and the displacement vector. Be sure to avoid mindlessly using *any 'ole angle* in the equation. A common physics lab involves applying a force to displace a cart up a ramp to the top of a chair or box. A *force* is applied to a cart to *displace* it up the incline at constant speed. Several incline angles are typically used; yet, the force is always applied parallel to the incline. The displacement of the cart is also parallel to the incline. Since F and d are in the same direction, the angle theta in the work equation is 0 degrees. Nevertheless, most students experienced the strong temptation to measure the angle of incline and use it in the equation. Don't forget: the angle in the equation is not just *any 'ole angle*. It is defined as the angle between the force and the displacement vector.

The Meaning of Negative Work

On occasion, a force acts upon a moving object to hinder a displacement. Examples might include a car skidding to a stop on a roadway surface or a baseball runner sliding to a stop on the infield dirt. In such instances, the force acts in the direction opposite the objects motion in order to slow it down. The force doesn't cause the displacement but rather *hinders* it. These situations involve what is commonly called *negative work*. The *negative* of negative work refers to the numerical value that results when values of F, d and theta are substituted into the work equation. Since the force vector is directly opposite the displacement vector, theta is 180 degrees. The cosine(180 degrees) is -1 and so a negative value results for the amount of work done upon the object. Negative work will become important (and more meaningful) in Lesson 2 as we begin to discuss the relationship between work and energy.

Units of Work

Whenever a new quantity is introduced in physics, the standard metric units associated with that quantity are discussed. In the case of work (and also energy), the standard metric unit is the Joule (abbreviated J). One Joule is equivalent to one Newton of force causing a displacement of one meter. In other words,

The Joule is the unit of work.

1 Joule = 1 Newton * 1 meter

1 J = 1 N * m

In fact, any unit of force times any unit of displacement is equivalent to a unit of work. Some nonstandard units for work are shown below. Notice that when analyzed, each set of units is equivalent to a force unit times a displacement unit.

Non-standard Units of Work:

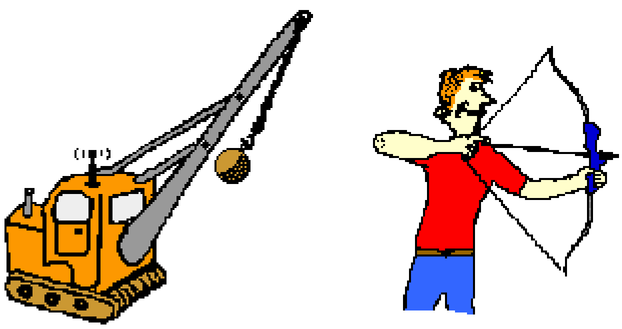
foot•pound   kg•(m/s^2)•m   kg•(m^2/s^2)

In summary, work is done when a force acts upon an object to cause a displacement. Three quantities must be known in order to calculate the amount of work. Those three quantities are force, displacement and the angle between the force and the displacement.

ENERGY

POTENTIAL ENERGY

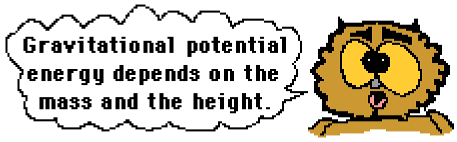
An object can store energy as the result of its position. For example, the heavy ball of a demolition machine is storing energy when it is held at an elevated position. This stored energy of position is referred to as potential energy. Similarly, a drawn bow is able to store energy as the result of its position. When assuming its *usual position* (i.e., when not drawn), there is no energy stored in the bow. Yet when its position is altered from its usual equilibrium position, the bow is able to store energy by virtue of its position. This stored energy of position is referred to as potential energy. Potential energy is the stored energy of position possessed by an object.



The massive ball of a demolition machine and the stretched bow possesses stored energy of position - potential energy.

Gravitational Potential Energy

The two examples above illustrate the two forms of potential energy to be discussed in this course - gravitational potential energy and elastic potential energy. Gravitational potential energy is the energy stored in an object as the result of its vertical position or height. The energy is stored as the result of the gravitational attraction of the Earth for the object. The gravitational potential energy of the massive ball of a demolition machine is dependent on two variables - the mass of the ball and the height to which it is raised. There is a direct relation between gravitational potential energy and the mass of an object. More massive objects have greater gravitational potential energy. There is also a direct relation between gravitational potential energy and the height of an object. The higher that an object is elevated, the greater the gravitational potential energy. These relationships are expressed by the following equation:



PE_{grav} = mass • g • height

PE_{grav} = m *• g • h

Derivation of Equation for P.E

Let the work done on the object against gravity = W

Work done, W = force × displacement

Work done, W = mg × h

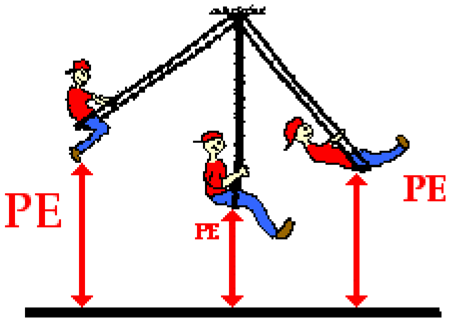
Work done, W = mgh

Since workdone on the object is equal to mgh, an energy equal to mgh units is gained by the object . This is the potential energy (Ep) of the object.

$$Ep = mgh$$

In the above equation, m represents the mass of the object, h represents the height of the object and g represents the gravitational field strength (9.8 N/kg on Earth) - sometimes referred to as the acceleration of gravity.

To determine the gravitational potential energy of an object, a *zero height position* must first be arbitrarily assigned. Typically, the ground is considered to be a position of zero height. But this is merely an arbitrarily assigned position that most people agree upon. Since many of our labs are done on tabletops, it is often customary to assign the tabletop to be the zero height position. Again this is merely arbitrary. If the tabletop is the zero position, then the potential energy of an object is based upon its height relative to the tabletop. For example, a pendulum bob swinging to and from above the tabletop has a potential energy that can be measured based on its height above the tabletop. By measuring the mass of the bob and the height of the bob above the tabletop, the potential energy of the bob can be determined.



Since the gravitational potential energy of an object is directly proportional to its height above the zero position, a *doubling* of the height will result in a *doubling* of the gravitational potential energy. A *tripling* of the height will result in a *tripling* of the gravitational potential energy.

Elastic Potential Energy

The second form of potential energy that we will discuss is elastic potential energy. Elastic potential energy is the energy stored in elastic materials as the result of their stretching or compressing. Elastic potential energy can be stored in rubber bands, bungee chords, trampolines, springs, an arrow drawn into a bow, etc. The amount of elastic potential energy

The compressed springs of a dart gun store elastic potential energy. When the trigger is pulled, the springs apply a force to do work on the dart.

stored in such a device is related to the amount of stretch of the device - the more stretch, the more stored energy.

Springs are a special instance of a device that can store elastic potential energy due to either compression or stretching. A force is required to compress a spring; the more compression there is, the more force that is required to compress it further. For certain springs, the amount of force is directly proportional to the amount of stretch or compression (x); the constant of proportionality is known as the spring constant (k).

$F_{spring} = k \cdot x$

Such springs are said to follow Hooke's Law. If a spring is not stretched or compressed, then there is no elastic potential energy stored in it. The spring is said to be at its *equilibrium position*. The equilibrium position is the position that the spring naturally assumes when there is no force applied to it. In terms of potential energy, the equilibrium position could be called the zero-potential energy position. There is a special equation for springs that relates the amount of elastic potential energy to the amount of stretch (or compression) and the spring constant. The equation is

$PE_{spring} = 0.5 \cdot k \cdot x^2$

where k = spring constant

x = amount of compression
(relative to equilibrium position)

To summarize, potential energy is the energy that is stored in an object due to its position relative to some zero position. An object possesses gravitational potential energy if it is positioned at a height above (or below) the zero height. An object possesses elastic potential energy if it is at a position on an elastic medium other than the equilibrium position.

KINETIC ENERGY

Kinetic energy is the energy of motion. An object that has motion - whether it is vertical or horizontal motion - has kinetic energy. There are many forms of kinetic energy - vibrational (the energy due to vibrational motion), rotational (the energy due to rotational motion), and translational (the energy due to motion from one location to another). To keep matters simple, we will focus upon translational kinetic energy. The amount of translational kinetic energy (from here on, the phrase kinetic energy will refer to translational kinetic energy) that an object has depends upon two variables: the mass (m) of the object and the speed (v) of the object. The following equation is used to represent the kinetic energy (KE) of an object.

$KE = 0.5 \cdot m \cdot v^2$

where m = mass of object

v = speed of object

Derivation of Equation for K.E

The relation connecting the initial velocity (u) and final velocity (v) of an object moving with a uniform acceleration a, and the displacement, S is

$$v2 - u2 = 2aS$$

This gives

$$S = v\ 2 - u\ 2\ 2a$$

We know F = ma. Thus using above equations, we can write the work done by the force, F as

$$W = ma \times v\ 2 - u\ 2\ 2a$$

or

$$W = 1\ 2\ m(\ v\ 2 - u\ 2\ )$$

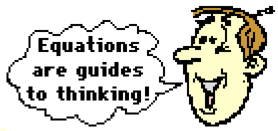
If object is starting from its stationary position, that is, u = 0, then

$$W = 1\ 2\ m\ v\ 2$$

It is clear that the work done is equal to the change in the kinetic energy of an object.

If u = 0, the work done will be W = 1 2 m v 2 .

Thus, the kinetic energy possessed by an object of mass, m and moving with a uniform velocity, v is Ek = ½ mv2



This equation reveals that the kinetic energy of an object is directly proportional to the square of its speed. That means that for a twofold increase in speed, the kinetic energy will increase by a factor of four. For a threefold increase in speed, the kinetic energy will increase by a factor of nine. And for a fourfold increase in speed, the kinetic energy will increase by a factor of sixteen. The kinetic energy is dependent upon the square of the speed. As it is often said, an equation is not merely a recipe for algebraic problem solving, but also a guide to thinking about the relationship between quantities.

Kinetic energy is a scalar quantity; it does not have a direction. Unlike velocity, acceleration, force, and momentum, the kinetic energy of an object is completely described by magnitude alone. Like work and potential energy, the standard metric unit of measurement for kinetic energy is the Joule. As might be implied by the above equation, 1 Joule is equivalent to 1 kg*(m/s)^2.

1 Joule = 1 kg • m^2/s^2

The Relation Between K.E and Momentum
Momentum is the quantity of motion of a moving body, its magnitude is equal to the producct of its mass and velocity of the body at a particular time.

If mass of the body = m and the velocity = v
its momentum (linear) p = mv

p = mv

Kinetic energy is defined as the energy possessed by a body because of its motion.

If mass of the boody = m
Velocity = v
Kinetic energy = ½ x mass x velocity2
⇒ K.E = ½ mv2
⇒ K.E = (½ mv) x v
but mv = p
⇒ K.E = ½ p x v
⇒ p = 2K.E/v
Or
Kinetic Energy = ½ mass x velocity2
⇒ K.E = ½ mv2

On multiplying and dividing the above equation with m
⇒ K.E = (½ mv)x(v) x m/m
⇒ K.E = ½ (mv x mv)/m
⇒ K.E = ½ (mv)2/m
⇒ K.E = ½ p2/m

Law of Conservation of Energy
The law of conservation of energy is the fundamental law, law of conservation of energy says that the energy can neither be created nor destroyed, the sum total energy existing in all forms in the universe remains constant. Energy can only be transformed from one form to another.

Principle of Conservation of Mechanical Energy, which states that the energy can neither be created nor destroyed; it can only be transformed from one state to another. Orthe total mechanical energy of a system is conserved if the forces doing the work on it are conservative.

Consider any two points A and B in the path of a body falling freely from a certain height H as in the above figure.

Total mechanical energy at A

$M.E_A = mgH + \frac{1}{2} mv_A^2$, here $v_A = 0$
$\Rightarrow M.E_A = mgH$

Total mechanical energy at B
$M.E_B = mg(H - h) + \frac{1}{2} mv_B^2$
$\Rightarrow M.E_B = mg(H - h) + \frac{1}{2} m (u^2 +2gh)$, where $u = 0$
$\Rightarrow M.E_B = mgH - mgh + \frac{1}{2} m (0^2 +2gh)$
$\Rightarrow M.E_B = mgH - mgh + \frac{1}{2} m \times 2gh$
$\Rightarrow M.E_B = mgH - mgh + mgh$
$\Rightarrow M.E_B = mgH$

Total mechanical energy at C
As the body reaches the ground its height from the ground becomes zero.

$M.E_C = mgH + \frac{1}{2} mv_C^2$, here $H = 0$
$\Rightarrow M.E_C = 0 + mv_C^2$, but v_C^2, $= 2gH$
$\Rightarrow M.E_C = \frac{1}{2} m \times 2gH$
$\Rightarrow M.E_C = mgH$

Hence the total mechanical energy at any point in its path is Constant i.e., $M.E_A = M.E_B = M.E_C = mgH$

According to the Principle of Conservation of Mechanical Energy, we can say that for points A and B, the total mechanical energy is constant in the path travelled by a body under the action of a conservative force, i.e., the total mechanical energy at A is equal to the total mechanical energy at B.

Note
Work done by a conservative force is path independent. It is equal to the difference between the potential energies of the initial and final positions and is completely recoverable."

As the work done by a conservative force depends on the initial and final position, we can say that work done by a conservative force in a closed path is zero as the initial and final positions in a closed path are the same.

Comparison between P.E and K.E

The energy possessed by a body or a system due to the motion of the body or of the particles in the system. Kinetic energy of an object is relative to other moving and stationary objects in its immediate environment.

Examples

Flowing water, such as when falling from a waterfall.

SI Unit

Joule (J)

Examples

Water at the top of a waterfall, before the precipice.

SI Unit

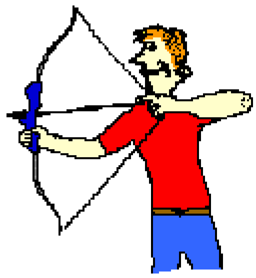
Joule (J)

- Power is the rate of work done.
- The commercial unit of energy is kilowatt hour (kW h).

1 kW h = 3.6×10^6 J

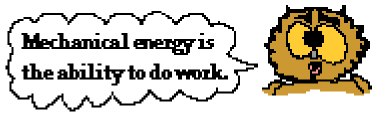
MECHANICAL ENERGY

work is done upon an object whenever a force acts upon it to cause it to be displaced. Work involves a force acting upon an object to cause a displacement. In all instances in which work is done, there is an object that supplies the force in order to do the work. If a World Civilization book is lifted to the top shelf of a student locker, then the student supplies the force to do the work on the book. If a plow is displaced across a field, then some form of farm equipment (usually a tractor or a horse) supplies the force to do the work on the plow. If a pitcher winds up and accelerates a baseball towards home plate, then the pitcher supplies the force to do the work on the baseball. If a roller coaster car is displaced from ground level to the top of the first drop of a roller coaster ride, then a chain driven by a motor supplies the force to do the work on the car. If a barbell is displaced from ground level to a height above a weightlifter's head, then the weightlifter is supplying a force to do work on the barbell. In all instances, an object that possesses some form of energy supplies the force to do the work. In the instances described here, the objects doing the work (a student, a tractor, a pitcher, a motor/chain) possess *chemical potential energy* stored in food or fuel that is transformed into work. In the process of doing work, the object that is doing the work exchanges energy with the object upon which the work is done. When the work is done upon the object, that object gains energy. The energy acquired by the objects upon which work is done is known as mechanical energy.


A drawn bow possesses mechanical energy in the form of elastic potential energy.

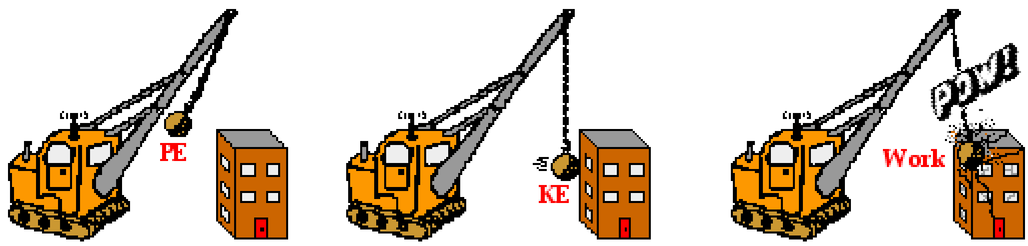
Mechanical energy is the energy that is possessed by an object due to its motion or due to its position. Mechanical energy can be either kinetic energy (energy of motion) or potential energy (stored energy of position). Objects have mechanical energy if they are in motion and/or if they are at some position relative to a *zero potential energy position* (for example, a brick held at a vertical position above the ground or zero height position). A moving car possesses mechanical energy due to its motion (kinetic energy). A moving baseball possesses mechanical energy due to both its high speed (kinetic energy) and its vertical position above the ground (gravitational potential energy). A World Civilization book at rest on the top shelf of a locker possesses mechanical energy due to its vertical position above the ground (gravitational potential energy). A barbell lifted high above a weightlifter's head possesses mechanical energy due to its vertical position above the ground (gravitational potential energy). A drawn bow possesses mechanical energy due to its stretched position (elastic potential energy).

Mechanical Energy as the Ability to Do Work



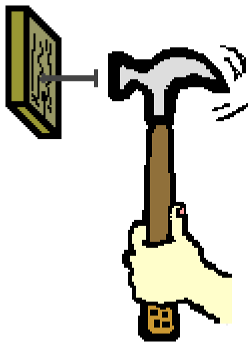
An object that possesses mechanical energy is able to do work. In fact, mechanical energy is often defined as the ability to do work. Any object that possesses mechanical energy - whether it is in the form of potential energy or kinetic energy - is able to do work. That is, its mechanical energy enables that object to apply a force to another object in order to cause it to be displaced.

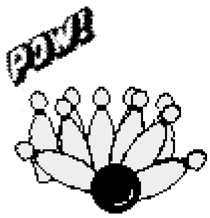
Numerous examples can be given of how an object with mechanical energy can harness that energy in order to apply a force to cause another object to be displaced. A classic example involves the massive wrecking ball of a demolition machine. The wrecking ball is a massive object that is swung backwards to a high position and allowed to swing forward into building structure or other object in order to demolish it. Upon hitting the structure, the wrecking ball applies a force to it in order to cause the wall of the structure to be displaced. The diagram below depicts the process by which the mechanical energy of a wrecking ball can be used to do work.



The massive ball of a demolition machine possesses mechanical energy - the ability to do work. When held at a height, it possesses mechanical energy in the form of potential energy. As it falls, it exhibits mechanical energy in the form of kinetic energy. As it strikes the structure to be demolished, it applies a force to displace the structure - i.e., it does work upon the structure.

A hammer is a tool that utilizes mechanical energy to do work. The mechanical energy of a hammer gives the hammer its ability to apply a force to a nail in order to cause it to be displaced. Because the hammer has mechanical energy (in the form of kinetic energy), it is able to do work on the nail. Mechanical energy is the ability to do work.



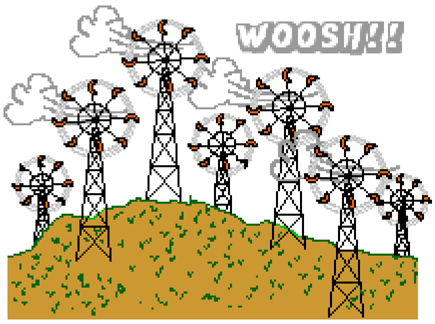


Another example that illustrates how mechanical energy is the ability of an object to do work can be seen any evening at your local bowling alley. The mechanical energy of a bowling ball gives the ball the ability to apply a force to a bowling pin in order to cause it to be displaced. Because the massive ball has mechanical energy (in the form of kinetic energy), it is able to do work on the pin. Mechanical energy is the ability to do work.

The compressed springs of a dart gun can apply a force to displace the dart.

A dart gun is still another example of how mechanical energy of an object can do work on another object. When a dart gun is loaded and the springs are compressed, it possesses mechanical energy. The mechanical energy of the compressed springs gives the springs the ability to apply a force to the dart in order to cause it to be displaced. Because of the springs have mechanical energy (in the form of elastic potential energy), it is able to do work on the dart. Mechanical energy is the ability to do work.

A common scene in some parts of the countryside is a "wind farm." High-speed winds are used to do work on the blades of a turbine at the so-called wind farm. The mechanical energy of the moving air gives the air particles the ability to apply a force and cause a displacement of the blades. As the blades spin, their energy is subsequently converted into electrical energy (a non-mechanical form of energy) and supplied to homes and industries in order to run electrical appliances. Because the moving wind has mechanical energy (in the form of kinetic energy), it is able to do work on the blades. Once more, mechanical energy is the ability to do work.



The kinetic energy of high speed winds contributes to its ability to do work.

The Total Mechanical Energy

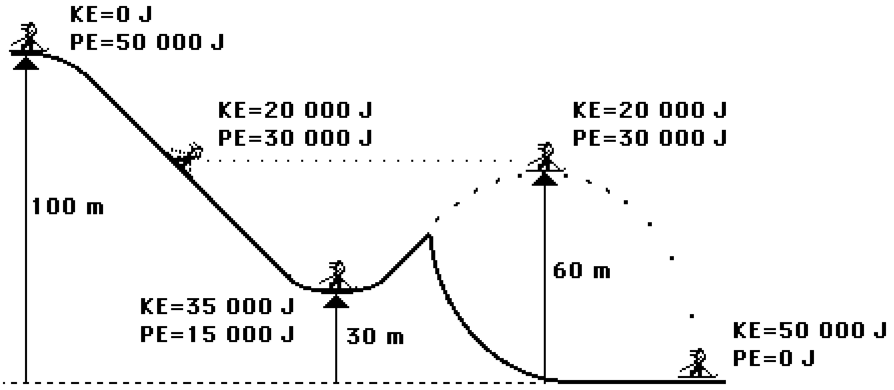
As already mentioned, the mechanical energy of an object can be the result of its motion (i.e., kinetic energy) and/or the result of its stored energy of position (i.e., potential energy). The total amount of mechanical energy is merely the sum of the potential energy and the kinetic energy. This sum is simply referred to as the total mechanical energy (abbreviated TME).

$TME = PE + KE$

As discussed earlier, there are two forms of potential energy discussed in our course - gravitational potential energy and elastic potential energy. Given this fact, the above equation can be rewritten:

$TME = PE_{grav} + PE_{spring} + KE$

The diagram below depicts the motion of Li Ping Phar (esteemed Chinese ski jumper) as she glides down the hill and makes one of her record-setting jumps.

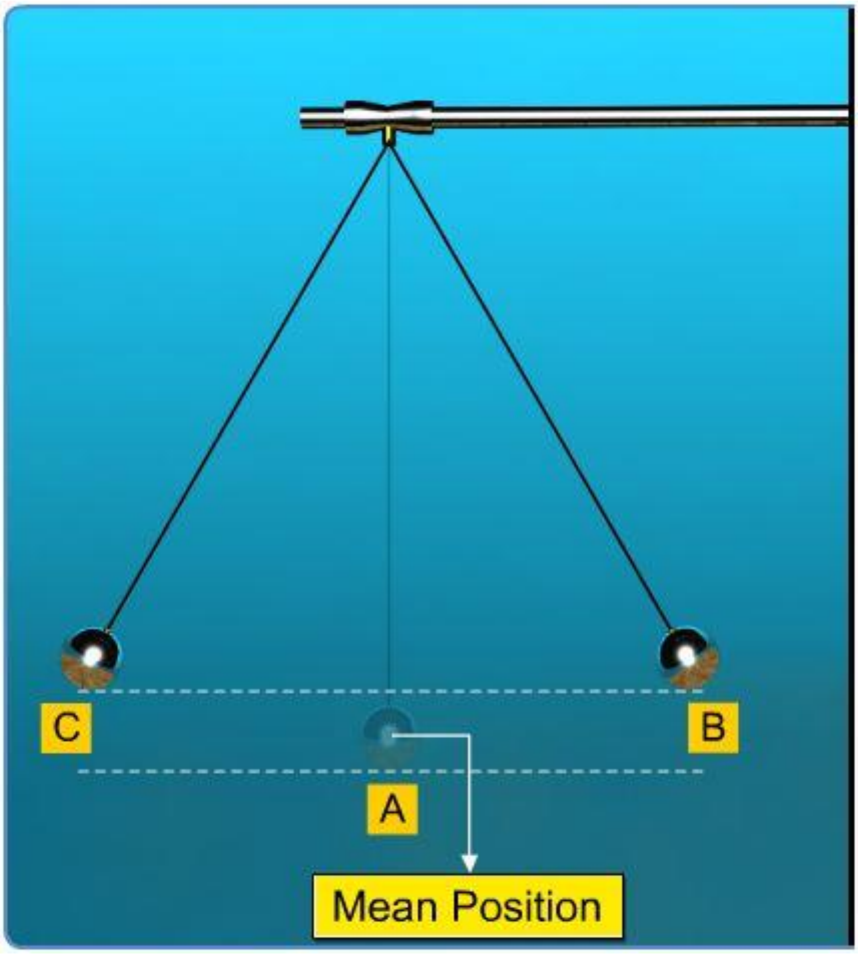


The total mechanical energy of Li Ping Phar is the sum of the potential and kinetic energies. The two forms of energy sum up to 50 000 Joules. Notice also that the total mechanical energy of Li Ping Phar is a constant value throughout her motion. There are conditions under which the total mechanical energy will be a constant value and conditions under which it will be a changing value.

Energy plays a major role in our daily day-to-day life. The two different major forms of mechanical energy are potential energy (P.E.) and kinetic energy (K.E.). Energy possessed by a body by virtue of the position from a reference level or arrangement is called its potential energy. The energy possessed by a body by virtue of its velocity is called its kinetic energy.

Conservation of Energy

Energy exists in several forms and can be converted from one form into another, but cannot be destroyed. This is called the conservation of energy.



The conversion of energy from potential energy into kinetic energy, and vice versa, can be illustrated using a simple pendulum.

- The simple pendulum is suspended freely from a stand. Note the initial position of the pendulum, at A.
- This is also called the reference position of the pendulum. Now draw the pendulum bob to one side, to a new position B, and release it.
- The bob goes from B to C, an extreme position on the other side of the reference position, and comes back to B, and so on.

When the bob is at extreme positions B and C, the velocity of the bob is zero. So its K.E. is also zero. However, the bob is at a height h above its mean position. Therefore, it has only potential energy at the extreme points. tAt the mean position, A, the potential energy of the bob is zero, because its height from the reference position, which again is A, is zero. It, thus, possesses only kinetic energy. The bob has only potential energy at the extreme positions, and only kinetic energy at the equilibrium position. In between these positions the bob possess both P.E. and K.E.

At any point in its oscillation, the total amount of mechanical energy that the bob possesses will be constant, it will actually be the sum of the P.E. and the K.E. Thus, the total energy in a system always remains constant, and this is the law of conservation of energy.

POWER

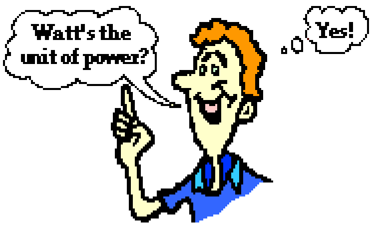
The quantity work has to do with a force causing a displacement. Work has nothing to do with the amount of time that this force acts to cause the displacement. Sometimes, the work is done very quickly and other times the work is done rather slowly. For example, a rock climber takes an abnormally long time to elevate her body up a few meters along the side of a cliff. On the other hand, a trail hiker (who selects the easier path up the mountain) might elevate her body a few meters in a short amount of time. The two people might do the same amount of work, yet the hiker does the work in considerably less time than the rock climber. The quantity that has to do with the rate at which a certain amount of work is done is known as the power. The hiker has a greater *power rating* than the rock climber.

Power is the rate at which work is done. It is the work/time ratio. Mathematically, it is computed using the following equation.

Power = Work / time

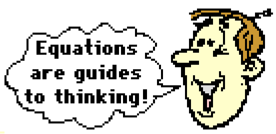
or

P = W / t

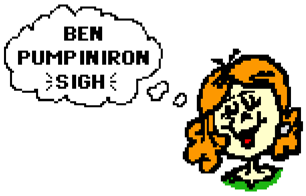


The standard metric unit of power is the Watt. As is implied by the equation for power, a unit of power is equivalent to a unit of work divided by a unit of time. Thus, a Watt is equivalent to a Joule/second. For historical reasons, the *horsepower* is occasionally used to describe the power delivered by a machine. One horsepower is equivalent to approximately 750 Watts.

Most machines are designed and built to do work on objects. All machines are typically described by a power rating. The power rating indicates the rate at which that machine can do work upon other objects. Thus, the power of a machine is the work/time ratio for that particular machine. A car engine is an example of a machine that is given a power rating. The power rating relates to how rapidly the car can accelerate the car. Suppose that a 40-horsepower engine could accelerate the car from 0 mi/hr to 60 mi/hr in 16 seconds. If this were the case, then a car with four times the horsepower could do the same amount of work in one-fourth the time. That is, a 160-horsepower engine could accelerate the same car from 0 mi/hr to 60 mi/hr in 4 seconds. The point is that for the same amount of work, power and time are inversely proportional. The power equation suggests that a more powerful engine can do the same amount of work in less time.



A person is also a machine that has a *power rating*. Some people are more power-full than others. That is, some people are capable of doing the same amount of work in less time or more work in the same amount of time. A common physics lab involves quickly climbing a flight of stairs and using mass, height and time information to determine a student's personal power. Despite the diagonal motion along the staircase, it is often assumed that the horizontal motion is constant and all the force from the steps is used to elevate the student upward at a constant speed. Thus, the weight of the student is equal to the force that does the work on the student and the height of the staircase is the upward displacement. Suppose that Ben Pumpiniron elevates his 80-kg body up the 2.0-meter stairwell in 1.8 seconds. If this were the case, then we could calculate Ben's *power rating*. It can be assumed that Ben must apply an 800-Newton downward force upon the stairs to elevate his body. By so doing, the stairs would push upward on Ben's body with just enough force to lift his body up the stairs. It can also be assumed that the angle between the force of the stairs on Ben and Ben's displacement is 0 degrees. With these two approximations, Ben's power rating could be determined as shown below.



$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{784\text{ N} \cdot 2.0\text{ m}}{1.8\text{ seconds}}$$

$$\text{Power} = 871\text{ Watts}$$

Ben's power rating is 871 Watts. He is quite a *horse*.

Another Formula for Power

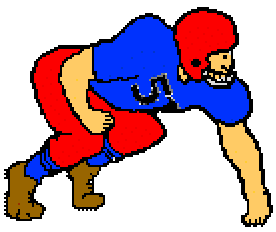
The expression for power is work/time. And since the expression for work is force*displacement, the expression for power can be rewritten as (force*displacement)/time. Since the expression for velocity is displacement/time, the expression for power can be rewritten once more as force*velocity. This is shown below.

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \cdot \text{Displacement}}{\text{Time}}$$

$$\text{Power} = \text{Force} \cdot \frac{\text{Displacement}}{\text{Time}}$$

$$\boxed{\text{Power} = \text{Force} \cdot \text{Velocity}}$$

This new equation for power reveals that a powerful machine is both strong (big force) and fast (big velocity). A powerful car engine is strong and fast. A powerful piece of farm equipment is strong and fast. A powerful weightlifter is strong and fast. A powerful lineman on a football team is strong and fast.



A powerful lineman is both STRONG (applies a big force) and FAST (displaces objects in small times).

A *machine* that is strong enough to apply a big force to cause a displacement in a small amount of time (i.e., a big velocity) is a powerful machine.