Current Electricity

Chapter-2: Current Electricity
Electric current; flow of electric charges in a metallic conductor; drift velocity; mobility and their relation with electric current; Ohm's law; electrical resistance; V-I characteristics (linear and non-linear), electrical energy and power; electrical resistivity and conductivity; Carbon resistors; colour code for carbon resistors; series and parallel combinations of resistors; temperature dependence of resistance.
Internal resistance of a cell; potential difference and emf of a cell; combination of cells in series and in parallel; Kirchhoff's laws and simple applications; Wheatstone bridge, metre bridge.
Potentiometer - principle and its applications to measure potential difference and for comparing EMF of two cells; measurement of internal resistance of a cell.

The branch of Physics which deals with the study of motion of electric charges is called current electricity. In an uncharged metallic conductor at rest, some (not all) electrons are continually moving randomly through the conductor because they are very loosely attached to the nuclei. The thermodynamic internal energy of the material is sufficient to liberate the outer electrons from individual atoms, enabling the electrons to travel through the material. But the net flow of charge at any point is zero. Hence, there is zero current. These are termed as free electrons. The external energy necessary to drive the free electrons in a definite direction is called electromotive force (emf). The emf is not a force, but it is the work done in moving a unit charge from one end to the other. The flow of free electrons in a conductor constitutes electric current.

2.1 Electric Current

1 ampere of electric current is produced when 1 coulomb of electric charge passes through the cross-section of the conductor in 1 second.

In a steady circuit, electrical potential at all points of a conductor remain constant with respect to time. In such a steady circuit, the amount of electric charge entering any cross-sectional area of a conductor in a given time interval is equal to the amount of electric charge leaving that cross-sectional area in the same interval of time. In other words, charge never accumulates at any point in the conductor. Also the electric charge is neither created nor destroyed at any point in the conductor. This means that electric charge is conserved.

The current is defined as the rate of flow of charges across any cross sectional area of a conductor. Let ΔQ amount of electric charge flow through any cross - sectional area of the conductor in time interval Δt. The average electric current flowing in time interval Δt is given by

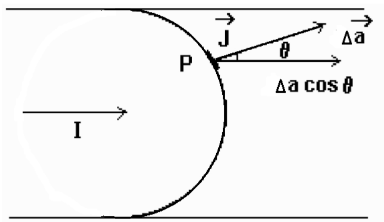
$$I = \frac{\Delta Q}{\Delta t}$$

The instantaneous electric current at time t is given by

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

Electric current is a fundamental quantity in S.I. unit and its symbol is A meaning ampere. Smaller units of the current are milliampere (mA = 10^{-3} A) and microampere (μ A = 10^{-6} A). Current is a scalar quantity. The direction of conventional current is taken as the direction of flow of positive charges or opposite to the direction of flow of electrons.

Current density



The electric current density near any point is defined as the amount of electric current flowing perpendicularly through the unit cross - sectional area near that point.

Let $\vec{J}$ be the electric current density at every point on the cross – sectional area, P be some point of the curved cross – sectional area of the conductor as shown in the figure,

$\Delta \vec{a}$ be the area vector of the surface near point P, component of which in the direction of the current is Δa cosθ, where θ is the angle between the area vector and the direction of the current.

Therefore, average current density near point P,

$J = \frac{\Delta I}{\Delta a \cos\theta}$ where ΔI is electric current flowing through the small area element near point P.

Therefore, electric current density,

$$J = \lim_{\Delta a \to 0} \frac{\Delta I}{\Delta a \cos\theta} = \frac{dI}{da\cos\theta}$$

$$\therefore dI = J da\cos\theta = \vec{J}.\overrightarrow{da}$$

$$\therefore \; I = \int dI = \int_a \vec{J}.\overrightarrow{da} = JA$$

$\therefore \; J = \frac{I}{A}$ where A = the entire cross - sectional area

Current density is a vector quantity. It is expressed in A m^{-2}

2.2 Electromotive Force and Terminal Voltage

The potential difference between the two poles of an electric cell is defined as the work done by the non-electrical force (due to chemical process taking place inside the cell) in moving a unit positive electric charge from a negative pole towards the positive pole. OR, terminal potential difference of a cell is defined as the potential difference between the two terminals of the cell in a closed circuit (i.e., when current is drawn from the cell)

The energy gained by the unit positive charge due to the non-electrical force, in moving it from the negative pole towards the positive pole, is called the emf (electromotive force) of the electric cell. Its unit is joule / coulomb (= volt). OR, the work done by the cell in moving positive charge through the whole circuit(including the cell) is called the electromotive force (emf) of the cell.

When the poles of the electric cell are connected externally by a conducting wire, current starts flowing in the circuit. The direction of the conventional current is from the positive pole to the negative pole of the cell in the conducting wire. Actually, electrons move from the negative pole to the positive pole of the cell in the conducting wire. Inside the cell, current is due to the movement of positive ions towards the positive pole and negative ions towards the negative pole of the cell. During the motion inside the cell, the charges have to overcome the resistance offered by the materials of the cell. This resistance is called the internal resistance, r, of the cell. Some of the energy gained by the positive charge due to the work done by the non-electrical force is dissipated as heat in overcoming the internal resistance of the cell. This reduces its overall energy in comparison with open circuit condition. Hence, the p.d. between the poles of the cell is slightly lower when current is flowing as compared to the open circuit condition. This reduced p.d. is called the terminal voltage of the cell.

The relation between the emf (Ɛ) and the terminal voltage (V) is

$$V = \mathcal{E} - Ir$$

Internal resistance of a cell

The electric current in an external circuit flows from the positive terminal to the negative terminal of the cell, through different circuit elements. In order to maintain continuity, the current has to flow through the electrolyte of the cell, from its negative terminal to positive terminal. During this process of flow of current inside the cell, a resistance is offered to current flow by the electrolyte of the cell. This is termed as the internal resistance of the cell.

A freshly prepared cell has low internal resistance and this increases with ageing.

<u>Determination of internal resistance of a cell using voltmeter</u>

The circuit connections are made as shown in Fig. With key K open, the emf of cell Ɛ is found by connecting a high resistance voltmeter across it. Since the high resistance voltmeter draws only a very feeble current for deflection, the circuit may be considered as an open circuit. Hence the voltmeter reading gives the emf of the cell. A small value of resistance R is included in the external circuit and key K is closed. The potential difference across R is equal to the potential difference across cell (V).

The potential drop across R, V = IR ...(1)

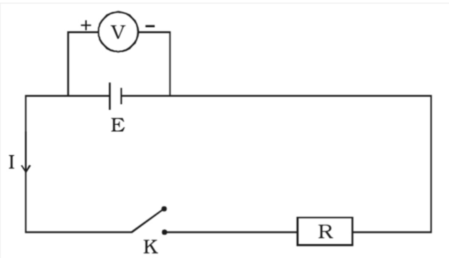
Due to internal resistance r of the cell, the voltmeter reads a value V, less than the emf of cell.

Then V = Ɛ – Ir or Ir = Ɛ−V ...(2)

Dividing equation (2) by equation (1)

$$r = \left(\frac{\varepsilon - V}{V}\right) R$$

Since Ɛ, V and R are known, the internal resistance r of the cell can be determined.



2.3 Ohm's Law

George Simon Ohm established the relationship between potential difference and current, which is known as Ohm's law. The current flowing through a conductor is

$$I = nAev_d$$

But $v_d = \frac{eE}{m} \cdot \tau$

$$\therefore I = nAe\frac{eE}{m} \cdot \tau = \frac{nAe^2\tau}{m} . E$$

$$\therefore I = \frac{nAe^2\tau}{mL} . V \qquad [\because V = E/L]$$

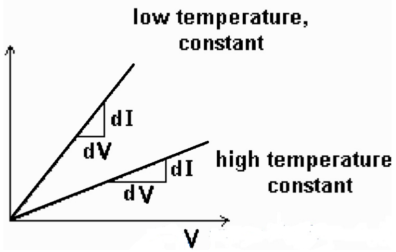
where V is the potential difference. The quantity mL/ nAe² τ is a constant for a given conductor, called electrical resistance (R).

$\therefore I \; \alpha \; V$

The law states that, at a constant temperature, the steady current flowing through a conductor is directly proportional to the potential difference between the two ends of the conductor.

(i.e) $I \; \alpha \; V$ or $I = \frac{1}{R} V$

$\therefore V = IR$ or $R \text{ (ohm)} = \frac{V\text{(volt)}}{I\text{(ampere)}}$



Resistance of a conductor is defined as the ratio of potential difference across the conductor to the current flowing through it. The unit of resistance is ohm (Ω)

The reciprocal of resistance (1 / R)is called the conductance of the material. Its unit is mho and symbol is Ʊ.

Metals, some insulators and many electrical components obey Ohm's law to perfection, but not all materials.

The graph of I vs. V will be a straight line for a conductor obeying Ohm's law at a constant temperature, but the resistance of the conductor changes with temperature. Also, Ohm's law holds good only when a steady current flows through a conductor.

2.4 Resistivity

Resistance (R) of a conductor varies directly as its length (l) and inversely as its cross - sectional area (A) at a given temperature.

$$\therefore R \propto l \text{ and } R \propto \frac{1}{a} \qquad \Rightarrow R \propto \frac{l}{a}$$

$$\therefore R = \rho \frac{l}{a}$$

Where, the constant ρ is known as the resistivity of the material. Its value depends on the type, temperature and the pressure existing on the given conductor. Its unit is ohm-meter (Wm).

Resistivity of the material of a conductor at a given temperature and pressure is defined as its resistance per unit length per unit cross-sectional area.

Conductivity is the inverse of resistivity. Its unit is mho-m^{-1} (or siemen - m^{-1}) and its symbol is σ (Ʊm^{-1})

Classification of materials in terms of resistivity

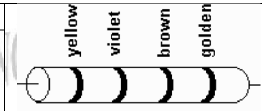
The resistivity of a material is the characteristic of that particular material. The materials can be broadly classified into conductors and insulators. The metals and alloys which have low resistivity of the order of 10^{-6} – 10^{-8} Ω m are good conductors of electricity. They carry current without appreciable loss of energy. Example : silver, aluminium, copper, iron, tungsten, nichrome, manganin, constantan. The resistivity of metals increases with increase in temperature. Insulators are substances which have very high resistivity of the order of 10^{8} – 10^{14} Ω m. They offer very high resistance to the flow of current and are termed non−conductors. Example : glass, mica, amber, quartz, wood, teflon, bakelite. In between these two classes of materials lie the semiconductors. They are partially conducting. The resistivity of semiconductor is 10^{-2} – 10^{4} Ω m. Example : germanium, silicon.

2.5 Colour Code for Carbon Resistors

There are two types of resistors: (i) wire wound and (ii) carbon resistors. Wire wound resistors are made of manganine, constantan and nichrome wires wound on a proper base. These resistors are expensive and huge in size. Hence, carbon resistors are used. Carbon resistor consists of a ceramic core, on which a thin layer of crystalline carbon is deposited. These resistors are cheaper, stable and small in size. Thus these resistors are widely used in electronic circuits.

The resistance of a carbon resistor is indicated by the colour code drawn on it (refer Table). A three colour code carbon resistor is discussed here. The silver or gold ring at one end corresponds to the tolerance. It is a tolerable range (±) of the resistance. The tolerance of silver and gold rings is 10% and 5% respectively. If there is no coloured ring at this end, the tolerance is 20%. The first two rings at the other end of tolerance ring are significant figures of resistance in ohm. The third ring indicates the powers of 10 to be multiplied or number of zeroes following the significant figure.

Colour	Digit	10^n	Colour	Digit	10^n	Tolerance
Black	0	10^0	Violet	7	10^7	
Brown	1	10^1	Grey	8	10^8	
Red	2	10^2	White	9	10^9	
Orange	3	10^3	Gold		10^{-1}	± 5 %
Yellow	4	10^4	Silver		10^{-2}	± 10 %
Green	5	10^5	No colour			± 20 %
Blue	6	10^6				
Slogan to memorize colour code - " B B ROY Goes to Bombay Via Gate-Way "						



The first band yellow has number 4 and the second band violet has number 7. This forms number 47. The third band brown means 10^1 as read from the third column of the table. 47×10^1 gives 470 Ω as the value of the resistor. The last band of golden colour gives tolerance of ± 5 %. Thus, true value of the resistor will be 470 ± 5 % Ω.

2.6 Origin of Resistivity

In vacuum, electrons have accelerated motion due to electric field. Contrary to this, electrons move with some average velocity due to electric field in a conductor under steady state of the flow of the current.

Outer orbit electrons of metals (known as valence electrons) are called free electrons as they are free to move in the entire space occupied by the piece of metal. The positively charged ions left behind oscillate about their lattice positions with energy depending upon temperature of metals. The free electrons move randomly in the space between the ions. In the absence of any electric field, the average number of electrons crossing any cross-section of a conductor being zero, there is no current. When potential difference is applied between the two ends of the conductor, electrons are dragged towards the positive end. If E is the electric field along the length of the conductor, an electron experiences a force eE opposite to the direction of the electric field as it is negatively charged and starts moving with an acceleration, a = eE / m. During this motion, electrons collide with constantly oscillating positive ions. With every such collision, a backward force acts on the electrons and they are slowly drifted with a constant average drift velocity v_d in a direction opposite to electric field, resulting in current.

If τ is the average time (known as relaxation time) between two successive collisions of the electron with the ions, then

$v_d = \frac{eE}{m}\tau$ ... (1)

Now, the current density J is directly proportional to E $\therefore J \propto E \Rightarrow J = \sigma E$ ... (2)

σ is called the conductivity of the material of conductor.

Also, If N = number of electrons crossing unit cross-sectional area of the conductor in 1 s, then

$J = Ne = n\,e\,v_d$ ... (3)

where n = number of free electrons per unit volume of the conductor = $\frac{N}{V_d}$

Putting the values of J and v_d from equations (2) and (1) in equation (3), we have

$\sigma E = ne\frac{eE}{m}\tau \qquad \therefore \sigma = \frac{ne^2\tau}{m} \qquad \therefore \rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$ ... (4)

With increase of temperature, ions oscillate faster resulting in decrease in relaxation time, τ, and increase in resistivity, ρ.

Vectorially, $\vec{J} = nq\vec{V_d}$. If q is positive, then $\vec{J}$ and $\vec{V_d}$ will have the same direction and if q is negative, then they will have opposite directions. Also, $\vec{J} = \frac{ne^2\tau}{m}\vec{E}$ or $\vec{J} = \sigma\vec{E}$

Mobility

The conductivity of any material is due to mobile charge carriers which can be electrons in a conductor or electrons and ions produced by ionization of air molecules or the positive and negative ions in electrolytes. In semi-conductors, electrons and holes (hole is the deficiency of electron in the covalent bond which behaves as a positively charged particle) are responsible for the flow of current.

Mobility, μ , of a charged particle is defined as its drift velocity per unit electric field intensity.

Thus, $\mu = \frac{V_d}{E}$

It takes the unit $m^2V^{-1}s^{-1}$

From equation, $nev_d = \sigma E$ for an electron moving in a conductor, we have $\frac{V_d}{E} = \frac{\sigma}{ne}$

$\therefore \mu = \frac{\sigma}{ne}$, where σ is the conductivity and n is the number of free electrons / unit volume.

Thus, for an electron, $\sigma_e = n_e e\mu_e$ and similarly for holes, $\sigma_h = n_h e\mu_h$

In a semi-conductor, the holes and electrons both constitute current in the same direction.

Hence, total conductivity,

$\sigma = \sigma_e + \sigma_h = n_e e\mu_e + n_h e\mu_h$

2.7 Temperature Dependence of Resistivity

The resistivity of metallic conductors increases with temperature as under. $\rho_\theta = \rho_0\,[\,1 + \alpha\,(\,\theta - \theta_0)\,]$ where, ρ_0 = resistivity at some reference temperature . ρ_θ= resisitivity at temperature. and α = temperature coefficient of resistivity and its unit is (°C $)^{-1}$.

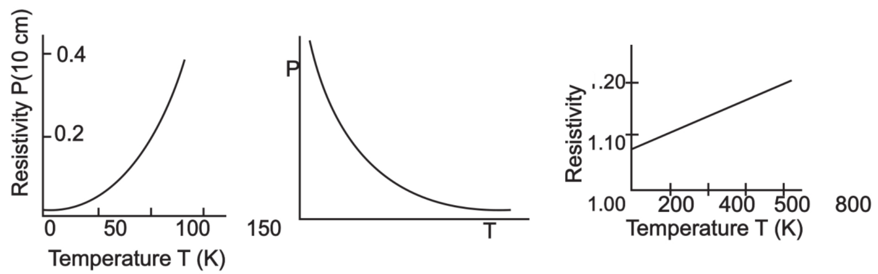
Ignoring variations in dimensions of the conductor, the above equation can also be written for the resistance of the conductor as follows: $R_\theta = R_0\,[\,1 + \alpha\,(\,\theta - \theta_0)\,]$

$\alpha = \frac{R_\theta - R_0}{R_0(\theta - \theta_0)}$

The temperature coefficient of resistance is defined as the ratio of increase in resistance per degree rise in temperature to its resistance at 0°C. Its unit is per °C.

Metals have positive temperature coefficient of resistance, i.e., their resistance increases with increase in temperature. Insulators and semiconductors have negative temperature coefficient of resistance, i.e., their resistance decreases with increase in temperature. A material with a negative temperature coefficient is called a thermistor. The temperature coefficient is low for alloys.

Variation of resistivity with temperature



(i) For conductor *(ii) For Ga As* *(iii) for nichrome (alloy)*

Nichrome (an alloy of nickel and chromium) has very high resistivity with low temperature dependence. Manganine (an alloy of copper, manganese and nickel) has resistivity which is almost constant with respect to the temperature. The resistivity of nichrome does not become zero even at absolute zero temperature, while the resistivity of pure metal becomes almost zero at absolute zero temperature which helps testing its purity.

Carbon, germanium, silicon and various other materials have negative value of α meaning that their resistivity decreases with temperature .

The relaxation time τ and to some extent the charged carrier density n, change with temperature in a semi-conductor and an insulator. The variation of n with temperature T is given by the equation

$n_T = n_0\,e^{-\frac{E_g}{K_B T}}$

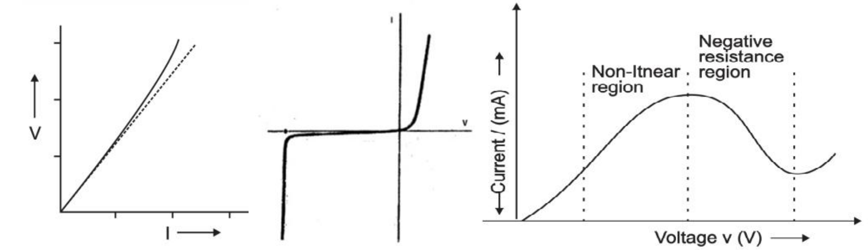
where E_g is the energy gap between the upper portion of the valence band and lower portion of the conduction band. k_B is the Boltzmann's constant. The equation suggests that for a semi-conductor, n increases with temperature thus increasing conductivity and reducing its resistivity.

2.8 Limitations of Ohm's Law

Although Ohm's law has been found valid over a large class of materials, there do exist materials and devices used in electric circuits where the proportionality of V and I does not hold, and they have the following V -I characteristics.

1. They are non-linear. V ceases to be proportional for higher value of I. Value of R increase as conductor gets heated up at higher value of current.
2. The absolute value of V depends on the polarity of V (i.e., it depends on its direction). In case of P-N junction magnitude of current depends on sign of potential difference applied.
3. V can have more than one value for a given value of current. The relation between V and I is not unique for the semiconductor Ga, As.

Variation of current with potential difference



(i) For conductor *(ii) For P-N junction diode* *(iii) For Ga As*

2.9 Superconductivity

"The resistance of certain materials becomes almost zero, when its temperature is lowered below certain fixed temperature (known as critical temperature T_c). The material in this situation is known as superconductor and this phenomenon is known as superconductivity."

Kamerlingh Onnes in 1911 showed that when the temperature of a specimen of mercury (Hg) is reduced to 4.3 K, its resistance reduces to 0.084 Ω and at 3 K, it becomes 3×10^{-6} Ω which is almost one lakhth part of its resistance at 0°C. Many metals and alloys exhibit the property of superconductivity. Si, Ge, Se and Te are some of the semi-conductors which behave as super-conductors under high pressure and low temperatures.

When current flows through a superconductor, its resistance being almost zero, practically no electrical energy is lost as heat and the current is sustained over a long time interval. The biggest hurdle in the commercial application of superconductors is the need for low temperature. Efforts are on by researchers all over the world to invent materials which can be used power can as superconductor be minimized.
at room temperature and thus transmission losses of electrical

Scientists Bardeen, Cooper and Schrieffer explained the phenomenon of superconductivity using quantum mechanics. Their theory is known as BCS theory. This theory is based on prediction of attraction between electrons under special circumstances. Physicists Bendnorz and Muller prepared compounds of copper oxides in 1986 having critical temperature of 30 K for which they were awarded Nobel Prize. Thereafter, critical temperature upto 135 K has been reached. Such super-conductors are known as high critical temperature super-conductors (HTS). HTS has applications in thin film devices, electric transmission over long distances, leviating trains (which can achieve speeds of 550 km/ h).

Applications of superconductors

(i) Superconductors form the basis of energy saving power systems, namely the superconducting generators, which are smaller in size and weight, in comparison with conventional generators.

(ii) Superconducting magnets have been used to levitate trains above its rails. They can be driven at high speed with minimal expenditure of energy.

(iii) Superconducting magnetic propulsion systems may be used to launch satellites into orbits directly from the earth without the use of rockets.

(iv) High efficiency ore–separating machines may be built using superconducting magnets which can be used to separate tumor cells from healthy cells by high gradient magnetic separation method.

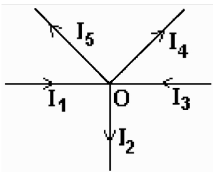
(v) Since the current in a superconducting wire can flow without any change in magnitude, it can be used for transmission lines.

(vi) Superconductors can be used as memory or storage elements in computers.

2.10 Electric Circuits and Kirchhoff's Rules

Kirchhoff's First Rule (current law):

"The algebraic sum of electric currents flowing through any junction is zero."



The convention is that, the current flowing towards a junction is positive and the current flowing away from the junction is negative.

Refer to the junction point O of the network as shown in the figure I_1, I_2, ..., I_5, are the currents meeting at the junction point O. Let Q_1, Q_2,..., Q_5 be the corresponding charges flowing in time t.

Then, $I_1 = \frac{Q_1}{t}$, $I_2 = \frac{Q_2}{t}$, ….., $I_5 = \frac{Q_5}{t}$

The total electric charge flowing towards the junction point = $Q_1 + Q_3$ and the total electric charge flowing out of the junction point = $Q_2 + Q_4 + Q_5$ in the same time.

Hence, by the law of conservation of electric charge,

$$Q_1 + Q_3 = Q_2 + Q_4 + Q_5$$

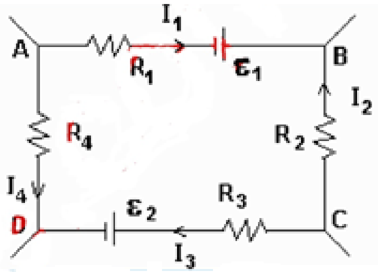
$$\therefore I_1t + I_3t = I_2t + I_4t + I_5t$$

$$\therefore I_1 + (-I_2) + I_3 + (-I_4) + (-I_5) = 0$$

It is clear from the above result that the algebraic sum of electric currents meeting at a junction is zero.

Kirchhoff's Second Rule (voltage law):

"The algebraic sum of the products of resistances and the corresponding currents flowing through them is equal to the algebraic sum of the emfs applied along the loop."



Kirchhoff's second rule is based on the conservation of energy and the concept of the electric potential.

Consider a closed path ABCDA formed using resistors R_1, R_2, R_3, R_4 and batteries having emfs $\mathcal{E}_1$ and $\mathcal{E}_2$.

The electric potential at any point in a steady circuit does not change with time. If VA is the electric potential at point A, then on measuring the electric potential at various points in the closed path ABCDA and coming back to A, the electric potential wouldagain be equal to V_A. This singular value of the electric potential is based on the law of conservation of energy.

Starting from A, as we move in a clockwise direction in the
This is because current flows from A to B which means that the end A of the resistor R_1 is at a higher potential. The potential increases by $\mathcal{E}_1$ as we move from negative terminal to positive terminal of battery having emf $\mathcal{E}_1$. Potential increases on going from point B to point C through the resistor R_2, as the direction of current is from C to B. Counting electric potential this way, on reaching the point A back, potential will be again V_A.

$$\therefore V_A - I_1R_1 + \mathcal{E}_1 + \quad I_2R_2 - I_3R_3 - \mathcal{E}_2 \; + I_4R_4 = V_A$$

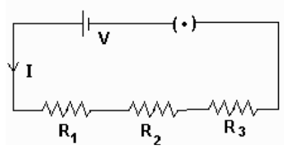
$$\therefore (- I_1R_1) + I_2R_2 + (- I_3R_3) + I_4R_4 = \mathcal{E}_2 - \mathcal{E}_1$$

In applying Kirchoff's laws to electrical networks, the direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the problems, the current will be found to have negative sign. If the result is positive, then the assumed direction is the same as actual direction.

It should be noted that, once the particular direction has been assumed, the same should be used throughout the problem. However, in the application of Kirchoff's second law, we follow that the current in clockwise direction is taken as positive and the current in anticlockwise direction is taken as negative.

2.11 Series and Parallel Connections of Resistors

Series Connection:



When resistors are connected end to end as shown in the figure, the same amount of current flows through them and they are said to have been connected in series. In this type of connection, potential difference across each resistor is different and is proportional to the value of the resistor.

Applying Kirchhoff's second rule, $I R_1 + I R_2 + I R_3 = V$

$\therefore$ I ($R_1 + R_2 + R_3$) = V

$\therefore$ $R_1 + R_2 + R_3$ = V / I ... ... (1)

If these three resistors are replaced by a single resistance R_s such that the same current I flows through the battery as earlier, then R_s is called the equivalent resistance of the three resistors connected in series.

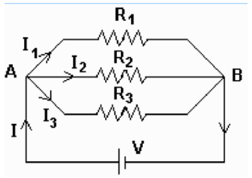
$\therefore$ R_s = V / I ... ... (2)

$\therefore$ $R_s = R_1 + R_2 + R_3$ [from equations (1) and (2)], and in general

$R_s = R_1 + R_2 + R_3 + ... + R_n$

The equivalent resistance of the resistors connected in series is greater than the greatest value of the resistors connected in series combination.

Parallel Connection:



When one of the ends of two or more resistors are connected to a single point and their other ends are connected to another common point in a circuit as shown in the figure, they are said to have been connected in parallel. In such a connection, the main line current distributes in the resistors, but the potential difference across them is the same.

Applying Kirchhoff's second rule to the loops, A - V - B - R_1 -A, A - V - B - R_2 - A and A - V - B - R3 - A,

$V = I_1 R_1 = I_2 R_2 = I_3 R_3$

$\therefore I_1 = \frac{V}{R_1}$ $I_2 = \frac{V}{R_2}$ $I_3 = \frac{V}{R_3}$

Applying Kirchhoff's first rule to the junction A,

$$I = I_1 + I_2 + I_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \quad \therefore \ \frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \quad ... (1)$$

If the parallel combination of resistors is replaced by a single resistor R_p whose value is such that the main line current I remains the same, then R_p is called the equivalent resistance of the three resistors connected in parallel. In that case,

$$V = I R_p \therefore \ \frac{I}{V} = \frac{1}{R_p} \quad ... ... (2)$$

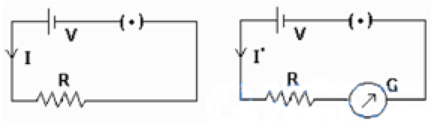
$$\therefore \ \frac{1}{R_p} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$ [from equations (1) and (2)], and in general

$$\frac{1}{R_p} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right)$$

The equivalent resistance of the resistors connected in parallel is less than the least value of the resistors connected in parallel combination.

2.12 Measurement of Voltage, Current and Resistance

Ammeter



Ammeter is used for measuring current. Galvanometer is a current sensing device which is used for this purpose. However, there are two practical difficulties in using galvanometer as an ammeter:

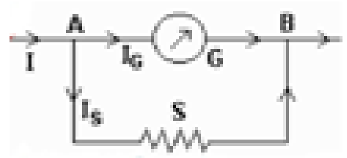
(1) If we want to measure the current flowing through the resistor R of the circuit shown in the figure, the current measuring instrument is to be connected in series with it as shown in the next figure. This adds resistance G of the instrument in the circuit thereby altering the current in the resistor from the original value I to I′.

(2) Further, a moving coil galvanometer is very sensitive and even a small current through it gives full scale deflection. Also the heat (I^2Gt) produced in it due to large current can damage its coil.

To overcome these problems, a low value resistance called shunt is connected in parallel to the galvanometer. As its value is much smaller than G, most of the current flows through it and the galvanometer

does not get damaged. Moreover, the equivalent resistance of the modified galvanometer is lower than that of the shunt which when connected in the circuit does not alter its resistance appreciably and hence the true value of the current is measured.

Formula of Shunt



Let I_G = maximum current capacity of the galvanometer,

I = desired maximum current in the ammeter

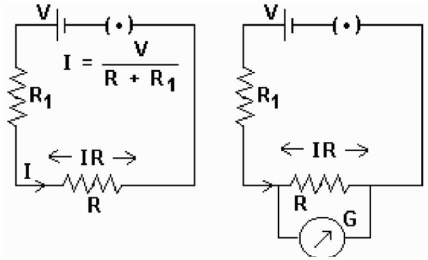
S = necessary value of the shunt

The value of the shunt should be so selected that the current I_G flows through the galvanometer and the balance I - I_G = I_s flows through the shunt as shown in the figure.

Applying Kirchhoff's second rule to the loop,

$- I_G G + (I - I_G) S = 0 \quad \therefore S = \frac{GI_G}{I - I_G}$

Voltmeter



Voltmeter is used to measure the potential difference between any two points of a circuit. Galvanometer with suitable modification is used for this purpose. However, there are two practical difficulties in using galvanometer as a voltmeter:

(1) To measure potential difference across a circuit element, galvanometer has to be connected in parallel to it. This alters the resistance of the circuit and the current flowing through the circuit element and hence the potential difference across it.

(2) Further, the galvanometer being a sensitive instrument, a large current can damage its coil.

To overcome these problems, a high value resistance called series resistance is connected in series with the galvanometer. If G is the resistance of the modified galvanometer, the new equivalent resistance of the circuit will be

$R' = R_1 + \frac{RG}{R+G}$ As G > > R, ignoring R, R + G = G

$\therefore R' = R_1 + R$

which is the same as the equivalent resistance of the original circuit.

Thus, resistance of the circuit is not changed much and as the value of G is large, most of the current flows through R which helps to measure correct value of IR. Moreover, very small current flows through the galvanometer due to its large resistance and is thus protected.

Deriving formula for the Series Resistance

Let I_G = maximum current capacity of the galvanometer,

G = resistance of galvanometer,

V = desired maximum voltage in the voltmeter

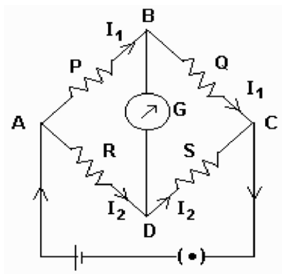
R_s = necessary value of the series resistance

Applying Ohm's law to the modified galvanometer as shown in the figure,

$I_G (G + R_s) = V$

$R_s = \frac{V}{I_G} - G$

Wheatstone Bridge



The network shown in the figure is known as Wheatstone bridge. It is a closed loop made up of four resistors P, Q, R and S. A source of emf (battery) is connected across AC and a galvanometer is connected across BD. Three of the four resistors are known whose values are so adjusted that the galvanometer shows zero deflection. In this condition, Wheatstone bridge is said to be balanced.

Applying Kirchhoff's second rule to the loop A - B - D - A under the balanced condition,

$- PI_1 + RI_2 = 0$. $PI_1 = RI_2$ ... ... (1)

Similarly, applying Kirchhoff's second rule to the loop B - C - D - B,

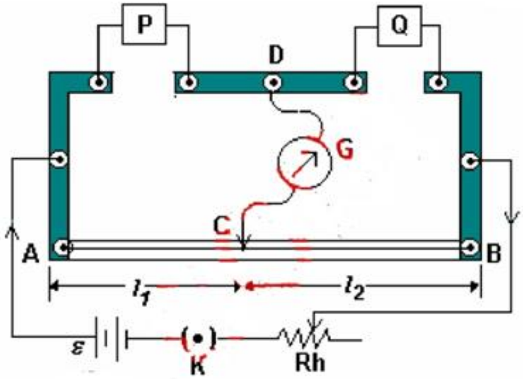
$- Q I_1 + S I_2 = 0$. $Q I_1 = S I_2$ ... ... (2)

Dividing equation (1) by equation (2), we have

$\frac{P}{Q} = \frac{R}{S}$ ... ... (3)

Hence by using the values of the three known resistors in the above equation, the value of the fourth unknown resistor can be calculated.

2.13 Metre bridge



Metre bridge is one form of W h e a t s t o n e ' s bridge, used in the laboratory to find the value of unknown resistance It consists a manganin or constantan wire AB of uniform diameter whose temperature coefficient is low, is used in place of resistors R and S. The wire is mounted on a wooden plank alongwith a meter rule. Copper strips are connected at the ends A and B of the wire as shown in the figure. The terminals on this strip are connected to a battery. Another copper strip is fixed between these two strips forming two gaps. In one gap, an unknown resistor P is connected and in the other a known resistor Q is connected. One end of a galvanometer is connected to the mid-point D of this strip and the other end to a jockey which can slide on the wire AB. For a given value Q, the jockey is slided on the wire in such a way that galvanometer shows zero deflection. If null point is obtained at C such that

AC = l_1 and CB = l_2, then from equation (3) above,

$\frac{P}{Q} = \frac{\rho l_1}{\rho l_2}$ $\therefore P = \frac{l_1}{l_2} Q$

The value of $l_1 : l_2$ is found out for different values of Q from which average value of P is calculated.

The specific resistance of the material of a wire is determined by knowing the resistance (P), radius (r) and length (L) of the wire using the expression $\rho = \frac{P\pi r^2}{L}$

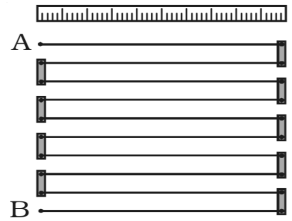
Though the connections between the resistances are made by thick copper strips of negligible resistance, and the wire AB is also soldered to such strips a small error will occur in the value of $\frac{l_1}{l_2}$ due to the end resistance. This error can be eliminated, if another set of readings are taken with P and Q interchanged and the average value of P is found.

2.14 Potentiometer

The terminal voltage of the battery is given by V = Ɛ - Ir.

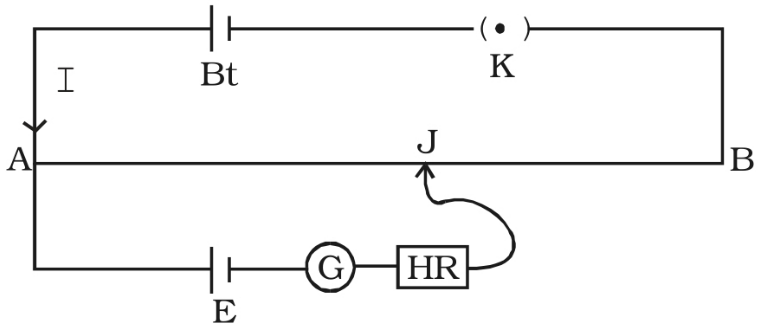
When we try to measure emf of the battery using a voltmeter, some current does flow through the battery. Hence, voltmeter measures terminal voltage V and not the emf Ɛ. If the term Ir in the above equation is zero, then only the voltmeter can measure emf of the battery. As internal resistance of the battery is not zero, this means that the current I must be zero. This is not possible in a voltmeter. Hence voltmeter cannot measure emf of the battery.

To measure emf of the battery, a device known as potentiometer is used. It consists of a ten metre long uniform wire of manganin or constantan stretched in ten segments, each of one metre length.



The segments are stretched parallel to each other on a horizontal wooden board. The ends of the wire are fixed to copper strips with binding screws. A metre scale is fixed on the board, parallel to the wire. Electrical contact with wires is established by pressing the jockey J.

Principle of potentiometer



A battery Bt is connected between the ends A and B of a potentiometer wire through a key K. A steady current I flows through the potentiometer wire (Fig). This forms the primary circuit. A primary cell is connected in series with the positive terminal A of the potentiometer, a galvanometer, high resistance and jockey. This forms the secondary circuit.

If the potential difference between A and J is equal to the emf of the cell, no current flows through the galvanometer. It shows zero deflection. AJ is called the balancing length. If the balancing length is l, the potential difference across AJ = Ixl where x is the resistance per unit length of the potentiometer wire and I the current in the primary circuit.

$\therefore$ E = Ixl,

since I and x are constants, E α l

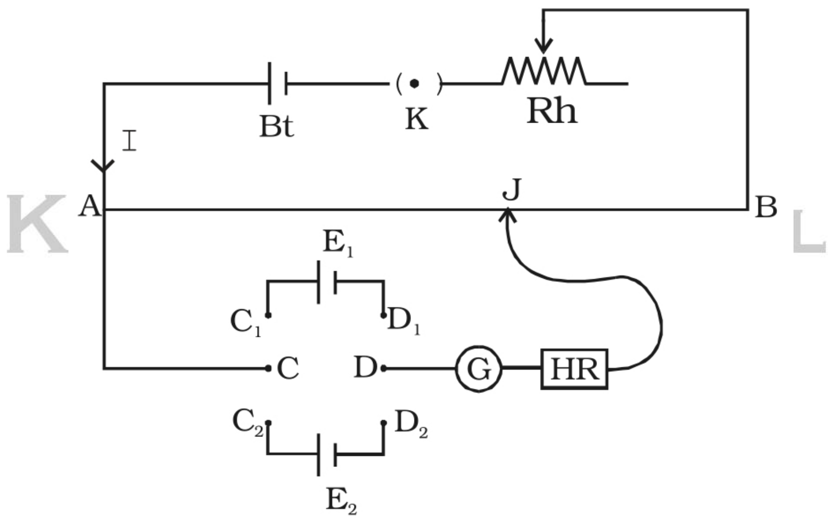
Hence emf of the cell is directly proportional to its balancing length. This is the principle of a potentiometer.

Comparison of emfs of two given cells using potentiometer

The potentiometer wire AB is connected in series with a battery (Bt), Key (K), rheostat (Rh) as shown in Fig. This forms the primary circuit. The end A of potentiometer is connected to the terminal C of a DPDT switch (six way key−double pole double throw). The terminal D is connected to the jockey (J) through a galvanometer (G) and high resistance (HR). The cell of emf E_1 is connected between terminals C_1 and D_1 and the cell of emf E_2 is connected between C_2 and D_2 of the DPDT switch.

Let I be the current flowing through the primary circuit and x be the resistance of the potentiometer wire per metre length.

The DPDT switch is pressed towards C_1, D_1 so that cell E_1 is included in the secondary circuit. The jockey



is moved on the wire and adjusted for zero deflection in galvanometer. The balancing length is l_1. The potential difference across the balancing length $l_1 = Ixl_l$. Then, by the principle of potentiometer,

$E_1 = Ixl_l$ ...(1)

The DPDT switch is pressed towards E_2. The balancing length l_2 for zero deflection in galvanometer is determined. The potential difference across the balancing length $l_2 = Ixl_2$, then

$E_2 = Ixl_2$ ...(2)

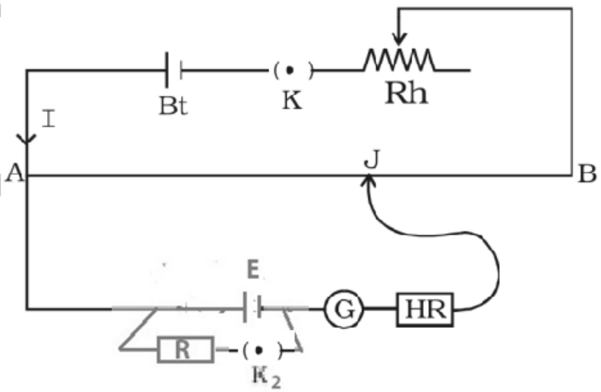
Dividing (1) and (2) we get

$\frac{E_1}{E_2} = \frac{l_1}{l_2}$

If emf of one cell (E_1) is known, the emf of the other cell (E_2) can be calculated using the relation.

$E_2 = E_1 \frac{l_1}{l_2}$

To measure internal resistance of a given cell using potentiometer



The potentiometer wire AB is connected in series with a battery (Bt), Key (K_1), rheostat (Rh) as shown in Fig. This forms the primary circuit. The end A of potentiometer is connected to the terminal C of a DPDT switch (six way key−double pole double throw). The terminal D is connected to the jockey (J) through a galvanometer (G) and high resistance (HR). A resistance box(R) and key(K_2) are connected across the cell. This complete the secondary circuit.

Let I be the current flowing through the primary circuit and x be the resistance of the potentiometer wire per metre length.

If the plug in key (K_2) is kept out and the jockey is moved on the wire and adjusted for zero deflection in galvanometer. The balancing length is l_1. The potential difference across the balancing length $l_1 = Ixl_l$. Then, by the principle of potentiometer,

$E = Ixl_l$ ...(1)

The resistance box (R) is now put in the plug key (K_2). The balancing length l_2 for zero deflection in galvanometer is determined. The potential difference (V) between the poles of the cell is given by

$V = Ixl_2$ ...(2)

Dividing (1) and (2) we get

$\frac{E}{V} = \frac{l_1}{l_2}$

The internal resistance of the cell is given by $r = \left(\frac{E}{V} - 1\right) R$

By substituting value of $\frac{E}{V}$ we get $r = \left(\frac{l_1}{l_2} - 1\right) R$

Where R is the resistance of the resistance box.

2.15 Comparison of emf and potential difference

1. The difference of potentials between the two terminals of a cell in an open circuit is called the electromotive force (emf) of a cell. The difference in potentials between any two points in a closed circuit is called potential difference.
2. The emf is independent of external resistance of the circuit, whereas potential difference is proportional to the resistance between any two points.

2.16 Electric energy and electric power.

If I is the current flowing through a conductor of resistance R in time t, then the quantity of charge flowing is, q = It. If the charge q, flows between two points having a potential difference V, then the work done in moving the charge is = V. q = V It.

Then, electric power is defined as the rate of doing electric work.

$\therefore$ Power $= \frac{Work\ done}{time} = \frac{VIt}{t}$ = VI

Electric power is the product of potential difference and current strength.

Since V = IR, Power = I^2R

Electric energy is defined as the capacity to do work. Its unit is joule. In practice, the electrical energy is measured by watt hour (Wh) or kilowatt hour (kWh). 1 kWh is known as one unit of electric energy.

(1 kWh = 1000 Wh = 1000 × 3600 J = 36 × 105 J)

2.17 Combination of Cells

There are three ways in which cells can be connected:

(1) Series connection, (2) Parallel connection and (3) Mixed connection.

Mixed Connection:

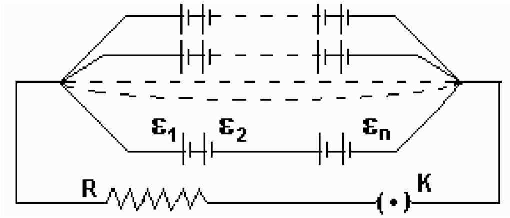
Let n = number of cells having emfs $\mathcal{E}_1, \mathcal{E}_2, .., \mathcal{E}_n$ and internal resistances $r_1, r_2, ...r_n$ connected in series and m = number of parallel connections of such series of cells.

For each series of cells, total emf is $\sum_{i=1}^{n} \mathcal{E}_i$ and total internal resistance is $\sum_{i=1}^{n} r_i$

For the mixed connection, the total emf is $\sum_{i=1}^{n} \varepsilon_i$ and net internal resistance is $\frac{1}{m}\sum_{i=1}^{n} r_i$

$\therefore$ Current I $= \frac{total\ emf}{total\ resistance}$

$= \frac{\sum_{i=1}^{n} \varepsilon_i}{R+\frac{1}{m}\sum_{i=1}^{n} r_i}$



If $\varepsilon_i = \varepsilon$ and $r_i = r$, then I $= \frac{mn\varepsilon}{mR+nr}$

Where R is the resistance in the circuit.

To determine how to arrange given total number of cells mn = x in the above mixed connection of cells for given external resistance R so that the current in the circuit becomes maximum,

We note that,

$I = \frac{mn\varepsilon}{(\sqrt{nr}-\sqrt{mR})^2+2\sqrt{mnrR}}$

which means that for current to be maximum,

nr = mR or R = nr / m, i.e., when the given number of cells are so arranged that their equivalent resistance equals the given fixed external resistance.

Solving mn = x ... (1) with nr = mR ... (2), we get $m = \sqrt{\frac{Xr}{R}}$ and $n = \sqrt{\frac{XR}{r}}$

and the value of the maximum current is

$I_{max} = \frac{\varepsilon}{2}\sqrt{\frac{X}{rR}}$

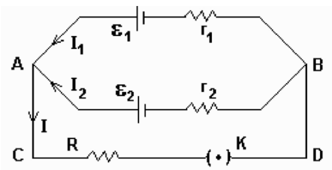
(Note: Important point to be noted here is that R is not variable. If R were to be variable, then the maximum current will be when R is zero, m and n are the variables whose values are to be obtained as above. If the above equations do not give integer values of m and n, then finding maximum current will involve examining several alternatives.)

<u>Series Connection</u>: For only one series of cells, m = 1, $I = \frac{\sum_{i=1}^{n} \varepsilon_i}{R+\sum_{i=1}^{n} r_i}$

<u>Parallel Connection</u>: For m parallel connections each having one cell, n = 1, $I = \frac{\varepsilon_i}{R+\frac{r_i}{m}}$

<u>Parallel Connection of Cells with different emfs and different internal resistances</u>:

Two cells having emfs ε_1 and ε_2 and internal resistances r_1 and r_2 are connected in parallel as shown in the figure.



Applying Kirchhoff's first rule at junction point A,

$I = I_1 + I_2$ ... ... (1)

Applying Kirchhoff's second rule to the loop ACDB ε_1A,

$I R + I_1 R_1 = \varepsilon_1$ $\quad \therefore I_1 = \frac{\varepsilon_1 - IR}{r_1}$ and similarly, $I_2 = \frac{\varepsilon_2 - IR}{r_2}$ ... ... (2)

Putting, values of I_1 and I_2 from equation (2) in equation (1),

$I = \frac{\varepsilon_1 - IR}{r_1} + \frac{\varepsilon_2 - IR}{r_2} \Rightarrow I = \frac{\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}}{1+\frac{R}{r_1}+\frac{R}{r_2}}$

If n cells are connected in parallel in the above arrangement, then $I = \frac{\sum_{i=1}^{n}\frac{\varepsilon_i}{r_i}}{1+R\sum_{i=1}^{n}\frac{1}{r_i}}$

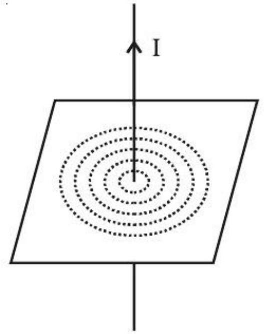
Magnetic Effects of Current and Magnetism

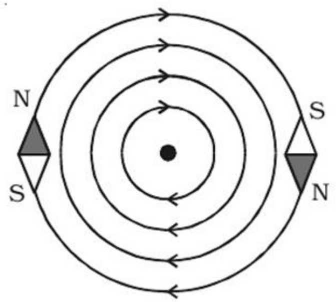
3.1 Magnetic effect of current

In 1820, Danish Physicist, Hans Christian Oersted observed that current through a wire caused a deflection in a nearby magnetic needle. This indicates that magnetic field is associated with a current carrying conductor.

3.2.1Magnetic field around a straight conductor carrying current

A smooth cardboard with iron filings spread over it, is fixed in a horizontal plane with the help of a clamp. A straight wire passes through a hole made at the centre of the cardboard (Fig 3.1).





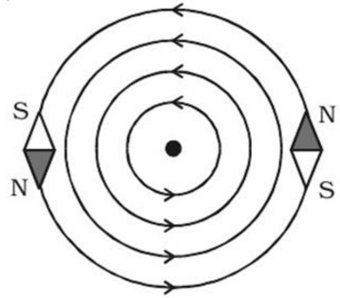


Fig 3.1 Magnetic field around a straight conductor carrying current *Fig 3.2 (a) Current inwards* *(b) Current Outwards*

A current is passed through the wire by connecting its ends to a battery. When the cardboard is gently tapped, it is found that the iron filings arrange themselves along concentric circles. This clearly shows that magnetic field is developed around a current carrying conductor.

To find the direction of the magnetic field, let us imagine, a straight wire passes through the plane of the paper and perpendicular to it. When a compass needle is placed, it comes to rest in such a way that its axis is always tangential to a circular field around the conductor. When the current is inwards (Fig 3.2a) the direction of the magnetic field around the conductor looks clockwise.

When the direction of the current is reversed, that it is outwards, (Fig 3.8b) the direction of the magnetic pole of the compass needle also changes showing the reversal of the direction of the magnetic field. Now, it is anticlockwise around the conductor.

This proves that the direction of the magnetic field also depends on the direction of the current in the conductor. This is given by Maxwell's rule.

Maxwells's right hand cork screw rule

If a right handed cork screw is rotated to advance along the direction of the current through a conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.

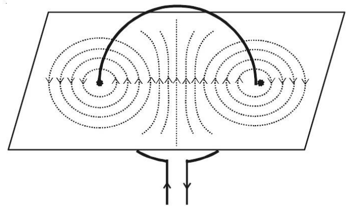


Fig 3.3 Magnetic field due to a circular loop carrying current

3.2.2Magnetic field due to a circular loop carrying current

A cardboard is fixed in a horizontal plane. A circular loop of wire passes through two holes in the cardboard as shown in Fig 3.3. Iron filings are sprinkled over the cardboard. Current is passed through the loop and the card board is gently tapped.

It is observed that the iron filings arrange themselves along the resultant magnetic field. The magnetic lines of force are almost circular around the wire where it passes through the cardboard. At the centre of the loop, the line of force is almost straight and perpendicular to the plane of the circular loop.

3.3 Biot – Savart Law

Biot and Savart conducted many experiments to determine the factors on which the magnetic field due to current in a conductor depends. The results of the experiments are summarized as Biot-Savart law.

Let us consider a conductor XY carrying a current I (Fig 3.4). AB = d l is a small element of the conductor. P is a point at a distance r from the midpoint O of AB. According to Biot and Savart, the magnetic induction dB at P due to the element of length d l is

(i) directly proportional to the current (I)

(ii) directly proportional to the length of the element (d l)

(iii) directly proportional to the sine of the angle between d l and the line joining element dl and the point P (sin θ)

(iv) inversely proportional to the square of the distance of the point from the element $\left(\frac{1}{r^2}\right)$

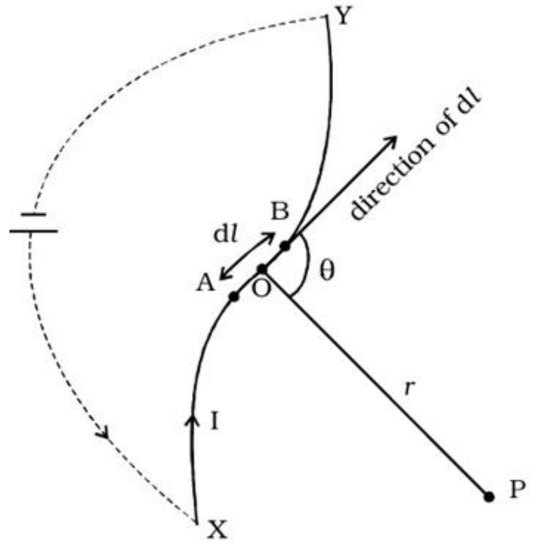


Fig 3.4 Biot - Savart Law

$\therefore$ dB α $\frac{Idl\sin\theta}{r^2}$

dB = k $\frac{Idl\sin\theta}{r^2}$, k is the constant of proportionality

The constant k = $\frac{\mu}{4\pi}$ where μ is the permeability of the medium

dB = $\frac{\mu}{4\pi}\frac{Idl\sin\theta}{r^2}$

$\mu = \mu_r \mu_o$ where μ_r is the relative permeability of the medium and μ_o is the permeability of free space.

$\mu_o = 4\pi \times 10^{-7}$ henry/metre. For air $\mu_r = 1$.

So, in air medium dB = $\frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$

In vector form, $\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}}{r^3}$ or $\frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\hat{r}}{r^2}$

The direction of dB is perpendicular to the plane containing current element Id l and r (i.e plane of the paper) and acts inwards. The unit of magnetic induction is tesla (or) weber m^{-2}.

3.3.1 Magnetic induction due to infinitely long straight conductor carrying current

XY is an infinitely long straight conductor carrying a current I (Fig 3.5). P is a point at a distance a from the conductor. AB is a small element of length dl. Θ is the angle between the current element I dl and the line joining the element dl and the point P.

According to Biot- Savart law, the magnetic induction at the point P due to the current element Id l is

dB = $\frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$ ...(1)

AC is drawn perpendicular to BP from A.

$\angle$OPA = ϕ , $\angle$APB = d ϕ

In Δ ABC, sin θ = $\frac{AC}{AB} = \frac{AC}{dl}$

$\therefore$ AC = d l sin θ ...(2)

From Δ APC, AC = rd ϕ ...(3)

From equations (2) and (3),

rd ϕ = d l sin θ ...(4)

substituting equation (4) in equation (1)

dB = $\frac{\mu_0}{4\pi}\frac{I\, rd\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{I\, d\phi}{r}$ ...(5)

In Δ OPA, cos ϕ = $\frac{a}{r}$

$\therefore$ r = $\frac{a}{\cos\phi}$ ...(6)

substituting equation (6) in equation (5)

dB = $\frac{\mu_0}{4\pi}\frac{I}{a}\cos\phi\, d\phi$

The total magnetic induction at P due to the conductor XY is

B = $\int_{-\phi_1}^{\phi_2} dB$

= $\int_{-\phi_1}^{\phi_2} \frac{\mu_0}{4\pi}\frac{I}{a}\cos\phi\, d\phi$

B = $\frac{\mu_o I}{4\pi a}[\sin\phi_1 + \sin\phi_2]$

For infinitely long conductor, $\phi_1 = \phi_2 = 90°$

$\therefore$ B = $\frac{\mu_0 I}{2\pi a}$

If the conductor is placed in a medium of permeability μ ,

B = $\frac{\mu I}{2\pi a}$

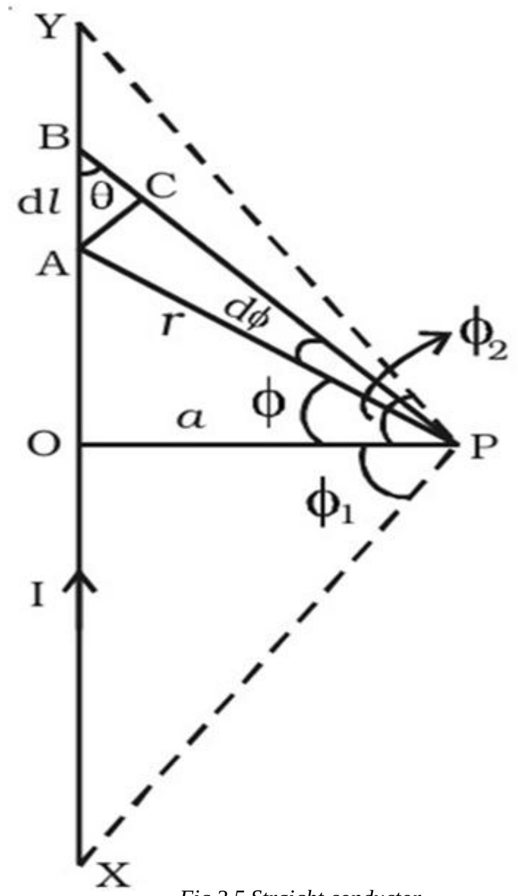


Fig 3.5 Straight conductor

3.3.2 Magnetic induction along the axis of a circular coil carrying current

Let us consider a circular coil of radius 'a' with a current I as shown in Fig 3.6.

P is a point along the axis of the coil at a distance x from the centre O of the coil. AB is an infinitesimally small element of length d l . C is the midpoint of AB and CP = r

According to Biot – Savart law, the magnetic induction at P due to the element d l is

dB = $\frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$, where θ is the angle between Id l and r

Here, θ = 90°

$\therefore$ dB = $\frac{\mu_0}{4\pi}\frac{Idl}{r^2}$

The direction of dB is perpendicular to the current element Id l and CP. It is therefore along PR perpendicular to CP.

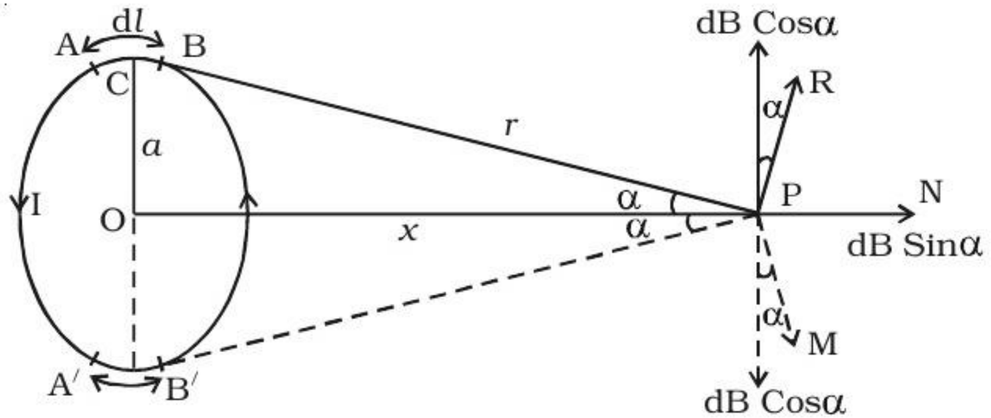


Fig. 3.6 Circular coil

Considering the diametrically opposite element A′B′, the magnitude of dB at P due to this element is the same as that for AB but its direction is along PM. Let the angle between the axis of the coil and the line joining the element (dl) and the point (P) be α.

dB is resolved into two components :- dB sin α along OP and dB cos α perpendicular to OP. dB cos α components due to two opposite elements cancel each other whereas dB sin α components get added up. So, the total magnetic induction at P due to the entire coil is

$$B = \int dB \sin\alpha = \int \frac{\mu_0}{4\pi}\frac{Idl}{r^2}\frac{a}{r} = \frac{\mu_0}{4\pi}\frac{Ia}{r^3}\int dl$$

$$= \frac{\mu_0}{4\pi}\frac{Ia}{r^3}2\pi a$$

$$= \frac{\mu_0 I a^2}{2(a^2+x^2)^{\frac{3}{2}}} \qquad (\because r^2 = a^2 + x^2)$$

If the coil contains n turns, the magnetic induction is

$$B = \frac{\mu_0 nIa^2}{2(a^2+x^2)^{\frac{3}{2}}}$$

At the centre of the coil, x = 0

$$B = \frac{\mu_0 nI}{2a}$$

3.3 Ampere's Circuital Law

Biot – Savart law expressed in an alternative way is called Ampere's circuital law.

The magnetic induction due to an infinitely long straight current carrying conductor is

$$B = \frac{\mu_0 I}{2\pi a}$$

$$B(2\pi a) = \mu_0 I$$

B (2π a) is the product of the magnetic field and the circumference of the circle of radius 'a' on which the magnetic field is constant. If L is the perimeter of the closed curve and I_o is the net current enclosed by the closed curve, then the above equation may be expressed as,

$$BL = \mu_0 I_o \qquad \text{....(1)}$$

In a more generalized way, Ampere's circuital law is written as

$$\oint \vec{B}.\vec{dl} = \mu_0 I_o \qquad \text{....(2)}$$

The line integral does not depend on the shape of the path or the position of the wire within the magnetic field. If the current in the wire is in the opposite direction, the integral would have the opposite sign. If the closed path does not encircle the wire (if a wire lies outside the path), the line integral of the field of that wire is zero. Although derived for the case of a number of long straight parallel conductors, the law is true for conductors and paths of any shape. Ampere's circuital law is hence defined using equation (1) as follows :

The line integral $\oint \vec{B}.\vec{dl}$ for a closed curve is equal to μ_0 times the net current Io through the area bounded by the curve.

3.3.1Solenoid

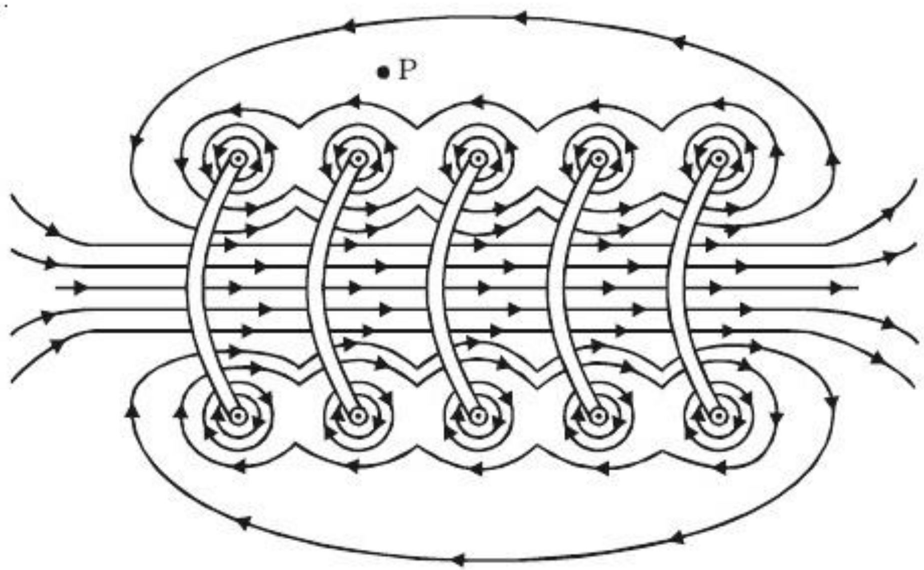


Fig 3.7 Magnetic field due to a current carrying solenoid

A long closely wound helical coil is called a solenoid. Fig 3.7 shows a section of stretched out solenoid. The magnetic field due to the solenoid is the vector sum of the magnetic fields due to current through individual turns of the solenoid. The magnetic fields associated with each single turn are almost concentric circles and hence tend to cancel between the turns.

At the interior midpoint, the field is strong and along the axis of the solenoid (i.e) the field is parallel to the axis. For a point such as P, the field due to the upper part of the solenoid turns tend to cancle the field due to the lower part of the solenoid turns, acting in opposite directions.

Hence the field outside the solenoid is nearly zero. The direction of the magnetic field due to circular closed loops (solenoid) is given by right hand palm-rule.

Right hand palm rule

The coil is held in the right hand so that the fingers point in the direction of the current in the windings. The extended thumb, points in the direction of the magnetic field. *Fig 3.8 Right hand palm rule*

3.3.2 Magnetic induction due to a long solenoid carrying current.

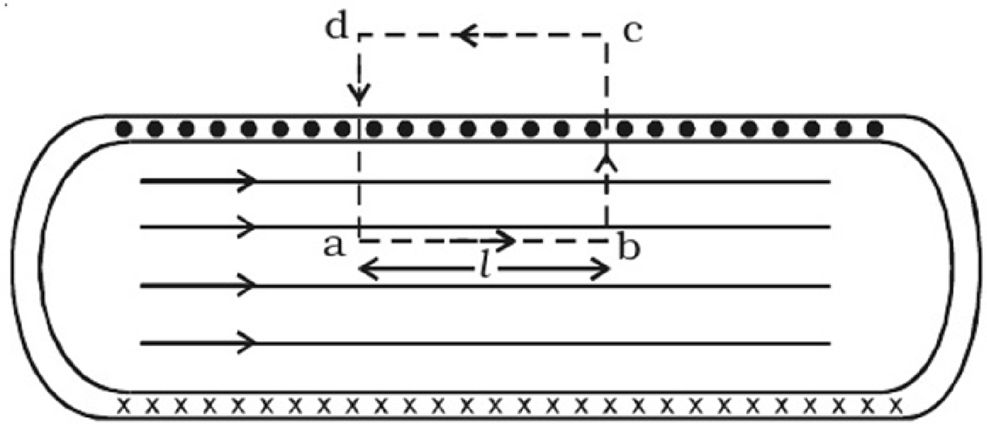


Fig 3.9 Magnetic field due to a long solenoid.

Let us consider an infinitely long solenoid having n turns per unit length carrying a current of I. For such an ideal solenoid (whose length is very large compared to its radius), the magnetic field at points outside the solenoid is zero.

A long solenoid appears like a long cylindrical metal sheet (Fig3.9).The upper view of dots is like a uniform current sheet coming out of the plane of the paper. The lower row of crosses is like a uniform current sheet going into the plane of the paper.

To find the magnetic induction (B) at a point inside the solenoid, let us consider a rectangular Amperean loop abcd. The line integral $\oint \vec{B}.\vec{dl}$ for the loop abcd is the sum of four integrals.

$$\therefore \oint \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl}$$

If l is the length of the loop, the first integral on the right side is B l . The second and fourth integrals are equal to zero because B is at right angles for every element dl along the path. The third integral is zero since the magnetic field at points outside the solenoid is zero.

$$\therefore \oint \vec{B}.\vec{dl} = B\, l \qquad ...(1)$$

Since the path of integration includes n l turns, the net current enclosed by the closed loop is

$$I_o = In\, l \qquad ...(2)$$

Ampere's circuital law for a closed loop is

$$\oint \vec{B}.\vec{dl} = \mu_o I_o \qquad ...(3)$$

Substituting equations (1) and (2) in equation (3)

$$B\, l = \mu_o In\, l$$

$$\therefore B = \mu_o nI \qquad ...(4)$$

The solenoid is commonly used to obtain uniform magnetic field.

By inserting a soft iron core inside the solenoid, a large magnetic field is produced

$$B = \mu nI = \mu_o \mu_r nI \qquad ...(5)$$

When a current carrying solenoid is freely suspended, it comes to rest like a suspended bar magnet pointing along north-south. The magnetic polarity of the current carrying solenoid is given by End rule.

End rule

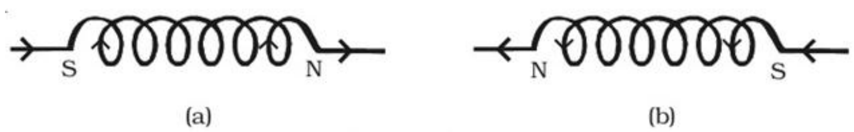


Fig 3.10

When looked from one end, if the current through the solenoid is along clockwise direction Fig 3.10a, the nearer end corresponds to south pole and the other end is north pole.

When looked from one end, if the current through the solenoid is along anti-clock wise direction, the nearer end corresponds to north pole and the other end is south pole (Fig 3.10b)

3.4 Magnetic Lorentz force

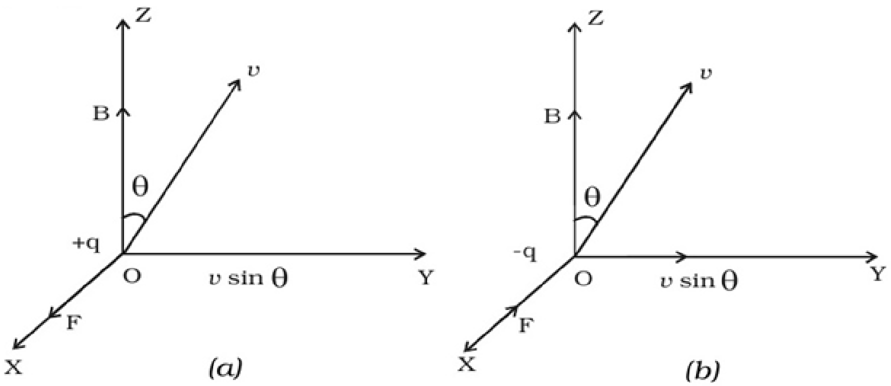


Fig 3.11

Let us consider a uniform magnetic field of induction B acting along the Z-axis. A particle of charge + q moves with a velocity v in YZ plane making an angle θ with the direction of the field (Fig 3.11a). Under the influence of the field, the particle experiences a force F.

H.A.Lorentz formulated the special features of the force F (Magnetic lorentz force) as under :

(i) the force F on the charge is zero, if the charge is at rest. (i.e) the moving charges alone are affected by the magnetic field.

(ii) the force is zero, if the direction of motion of the charge is either parallel or anti-parallel to the field and the force is maximum, when the charge moves perpendicular to the field.

(iii) the force is proportional to the magnitude of the charge (q)

(iv) the force is proportional to the magnetic induction (B)

(v) the force is proportional to the speed of the charge (v)

(vi) the direction of the force is oppositely directed for charges of opposite sign (Fig 3.11b).

All these results are combined in a single expression as

$$\vec{F} = q(\vec{v} x \vec{B})$$

The magnitude of the force is

$$F = Bqv \sin\, \theta$$

Since the force always acts perpendicular to the direction of motion of the charge, the force does not do any work.

In the presence of an electric field E and magnetic field B, the total force on a moving charged particle is

$$\vec{F} = q[(\vec{v} x \vec{B}) + \vec{E}]$$

3.4.1 Motion of a charged particle in a uniform magnetic field.

Let us consider a uniform magnetic field of induction B acting along the Z-axis. A particle of charge q and mass m moves in XY plane.

At a point P, the velocity of the particle is v. (Fig 3.12)

The magnetic lorentz force on the particle is $\vec{F} = q(\vec{v} \times \vec{B})$. Hence $\vec{F}$ acts along PO perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.

Since the force acts perpendicular to its velocity, the force does not do any work. So, the magnitude of the velocity remains constant and only its direction changes. The force F acting towards the point O acts as the centripetal force and makes the particle to move along a circular path. At points Q and R, the particle experiences force along QO and RO respectively.

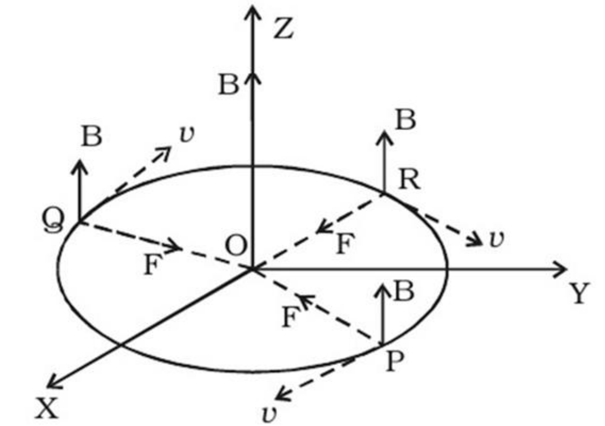


Fig 3.12 Motion of a charged particle

Since $\vec{v}$ and $\vec{B}$ are at right angles to each

$$F = Bqv \sin 90° = Bqv$$

This magnetic lorentz force provides the necessary centripetal force.

$$Bqv = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq} \qquad ...(1)$$

It is evident from this equation, that the radius of the circular path is proportional to (i) mass of the particle and (ii) velocity of the particle.

From equation (1), $\frac{v}{r} = \frac{Bq}{m}$

$$\omega = \frac{Bq}{m} \qquad ...(2)$$

This equation gives the angular frequency of the particle inside the magnetic field. Period of rotation of the particle,

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi m}{Bq} \qquad ...(3)$$

From equations (2) and (3), it is evident that the angular frequency and period of rotation of the particle in the magnetic field do not depend upon (i) the velocity of the particle and (ii) radius of the circular path.

3.4.2 Cyclotron

Cyclotron is a device used to accelerate charged particles to high energies. It was devised by Lawrence.

Principle

Cyclotron works on the principle that a charged particle moving normal to a magnetic field experiences magnetic lorentz force due to which the particle moves in a circular path.

Construction

It consists of a hollow metal cylinder divided into two sections D_1 and D_2 called Dees, enclosed in an evacuated chamber (Fig 3.13). The Dees are kept separated and a source of ions is placed at the centre in the

gap between the Dees. They are placed between the pole pieces of a strong electromagnet. The magnetic field acts perpendicular to the plane of the Dees. The Dees are connected to a high frequency oscillator.

Working

When a positive ion of charge q and mass m is emitted from the source, it is accelerated towards the Dee having a negative potential at that instant of time. Due to the normal magnetic field, the ion experiences magnetic lorentz force and moves in a circular path. By the time the ion arrives at the gap between the Dees, the polarity of the Dees gets reversed. Hence the particle is once again accelerated and moves into the other Dee with a greater velocity along a circle of greater radius. Thus the particle moves in a spiral path of increasing radius and when it comes near the edge, it is taken out with the help of a deflector plate (D.P). The particle with high energy is now allowed to hit the target T. When the particle moves along a circle of radius r with a velocity v, the magnetic Lorentz force provides the necessary centripetal force.

$$Bqv = \frac{mv^2}{r}$$

$$\therefore \frac{v}{r} = \frac{Bq}{m} = \text{constant} \qquad ...(1)$$

The time taken to describe a semi-circle

$$t = \frac{\pi r}{v} \qquad ...(2)$$

Substituting equation (1) in (2),

$$t = \frac{\pi m}{Bq} \qquad ...(3)$$

It is clear from equation (3) that the time taken by the ion to describe a semi-circle is independent of

[i] the radius (r) of the path and [ii] the velocity (v) of the particle

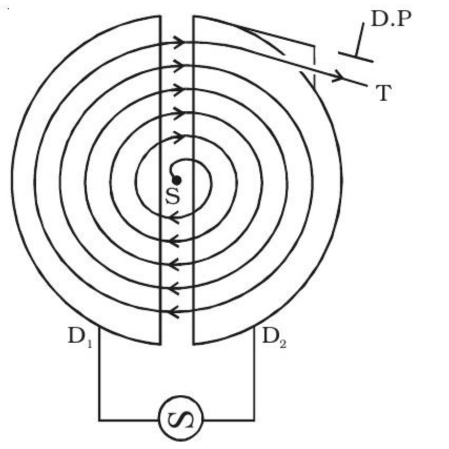


Fig 3.13 Cyclotron

Hence, period of rotation T = 2t

$$\therefore T = \frac{2\pi m}{Bq} = \text{constant} \qquad ...(4)$$

So, in a uniform magnetic field, the ion traverses all the circles in exactly the same time. The frequency of rotation of the particle,

$$\upsilon = \frac{1}{T} = \frac{Bq}{2\pi m} \qquad ...(5)$$

If the high frequency oscillator is adjusted to produce oscillations of frequency as given in equation (5), resonance occurs.

Cyclotron is used to accelerate protons, deutrons and α - particles.

Limitations

(i) Maintaining a uniform magnetic field over a large area of the Dees is difficult.
(ii) At high velocities, relativistic variation of mass of the particle upsets the resonance condition.
(iii) At high frequencies, relativistic variation of mass of the electron is appreciable and hence electrons cannot be accelerated by cyclotron.

3.5 Force on a current carrying conductor placed in a magnetic field.

Let us consider a conductor PQ of length l and area of cross section A. The conductor is placed in a uniform magnetic field of induction B making an angle θ with the field [Fig 3.14]. A current I flows along PQ. Hence, the electrons are drifted along QP with drift velocity v_d. If n is the number of free electrons per unit volume in the conductor, then the current is

$$I = nAv_d e$$

Multiplying both sides by the length l of the conductor,

$$\therefore I\,l = nAv_d\,e\,l$$

Therefore the current element,

$$I\vec{l} = -\,nA\vec{v}_d\,e\,l \quad ...(1)$$

The negative sign in the equation indicates that the direction of current is opposite to the direction of drift velocity of the electrons. Since the electrons move under the influence of magnetic field, the magnetic lorentz force on a moving electron.

$$\vec{f} = -\,e\,(\vec{v}_d \times \vec{B}) \quad ...(2)$$

The negative sign indicates that the charge of the electron is negative.

The number of free electrons in the conductor

$$N = nA\,l \quad ...(3)$$

The magnetic lorentz force on all the moving free electrons

$$\vec{F} = N\vec{f}$$

Substituting equations (2) and (3) in the above equation

$$\vec{F} = nA\,l\,\{-e\,(\vec{v}_d \times \vec{B})\}$$

$$\vec{F} = -\,nA\,l\,\{e\,(\vec{v}_d \times \vec{B})\} \quad ...(4)$$

Substituting equation (1) in equation (4)

$$\vec{F} = I\,\vec{l} \times \vec{B}$$

This total force on all the moving free electrons is the force on the current carrying conductor placed in the magnetic field.

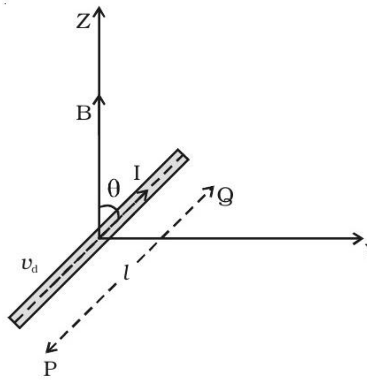


Fig 3.14 Force on a current carrying conductor placed in a magnetic field

Magnitude of the force

The magnitude of the force is $F = BI\,l\sin\theta$

(i) If the conductor is placed along the direction of the magnetic field, $\theta = 0°$, Therefore force $F = 0$.

(ii) If the conductor is placed perpendicular to the magnetic field, $\theta = 90°$, $F = BI\,l$. Therefore the conductor experiences maximum force.

Direction of force

The direction of the force on a current carrying conductor placed in a magnetic field is given by Fleming's Left Hand Rule.

The forefinger, the middle finger and the thumb of the left hand are stretched in mutually perpendicular directions. If the forefinger points in the direction of the magnetic field, the middle finger points in the direction of the current, then the thumb points in the direction of the force on the conductor.

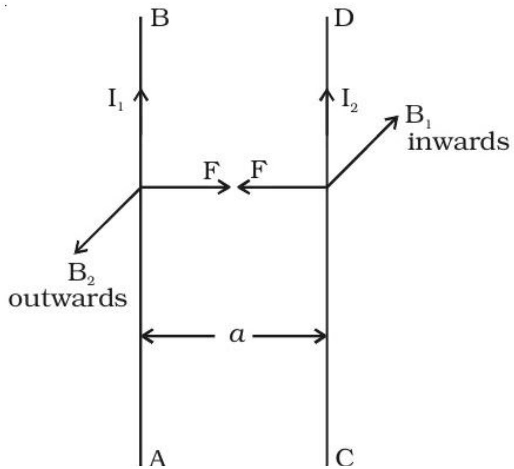


Fig. 3.15 Force between two long parallel current-carrying conductors

3.5.1 Force between two long parallel current-carrying conductors

AB and CD are two straight very long parallel conductors placed in air at a distance a. They carry currents I_1 and I_2 respectively. (Fig 3.15). The magnetic induction due to current I_1 in AB at a distance a is

$$B_1 = \frac{\mu_0 I_1}{2\pi a} \quad ....(1)$$

This magnetic field acts perpendicular to the plane of the paper and inwards.The conductor CD with current I_2 is situated in this magnetic field.

Hence, force on a segment of length l of CD due to magnetic field B_1 is

$$F = B_1 I_2\,l$$

substituting equation (1)

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi a} \quad ....(2)$$

By Fleming's Left Hand Rule, F acts towards left. Similarly, the magnetic induction due to current I_2 flowing in CD at a distance a is

$$B_2 = \frac{\mu_0 I_2}{2\pi a} \quad ...(3)$$

This magnetic field acts perpendicular to the plane of the paper and outwards. The conductor AB with current I_1, is situated in this field. Hence force on a segment of length l of AB due to magnetic field B_2 is

$$F = B_2 I_1\,l$$

substituting equation (3)

$$\therefore F = \frac{\mu_0 I_1 I_2 l}{2\pi a} \quad ...(4)$$

By Fleming's left hand rule, this force acts towards right. These two forces given in equations (2) and (4) attract each other. Hence, two parallel wires carrying currents in the same direction attract each other and if they carry currents in the opposite direction, repel each other.

Definition of ampere

The force between two parallel wires carrying currents on a segment of length l is

$$F = \frac{\mu_0 I_1 I_2}{2\pi a}\,l$$

$\therefore$ Force per unit length of the conductor is

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

If $I_1 = I_2 = 1A$, a = 1m

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\,\frac{1\times 1}{1} = \frac{4\pi\times 10^{-7}}{2\pi} = 2\times 10^{-7}\ Nm^{-1}$$

The above conditions lead the following definition of ampere.

Ampere is defined as that constant current which when flowing through two parallel infinitely long straight conductors of negligible cross section and placed in air or vacuum at a distance of one metre apart, experience a force of 2×10^{-7} newton per unit length of the conductor.

3.6 Torque experienced by a current loop in a uniform magnetic field

Let us consider a rectangular loop PQRS of length l and breadth b (Fig 3.16). It carries a current of I along PQRS. The loop is placed in a uniform magnetic field of induction B. Let θ be the angle between the normal to the plane of the loop and the direction of the magnetic field.

Force on the arm QR, $\vec{F_1} = I\,\overrightarrow{QR} \times \vec{B}$

Since the angle between $\overrightarrow{QR}$ and $\vec{B}$ is $(90° - \theta)$,

Magnitude of the force $F_1 = BIb\sin(90° - \theta)$

ie. $F_1 = BIb\cos\theta$

Force on the arm SP, $\vec{F_2} = I\,\overrightarrow{SP} \times \vec{B}$

Since the angle between $\overrightarrow{SP}$ and $\vec{B}$ is $(90° + \theta)$,

Magnitude of the force $F_2 = BIb\cos\theta$

The forces F_1 and F_2 are equal in magnitude, opposite in direction and have the same line of action. Hence their resultant effect on the loop is zero.

Force on the arm PQ, $\vec{F_3} = I\,\vec{PQ} \times \vec{B}$

Since the angle between $\vec{PQ}$ and $\vec{B}$ is 90°,

Magnitude of the force $F_3 = BI\,l \sin 90° = BI\,l$

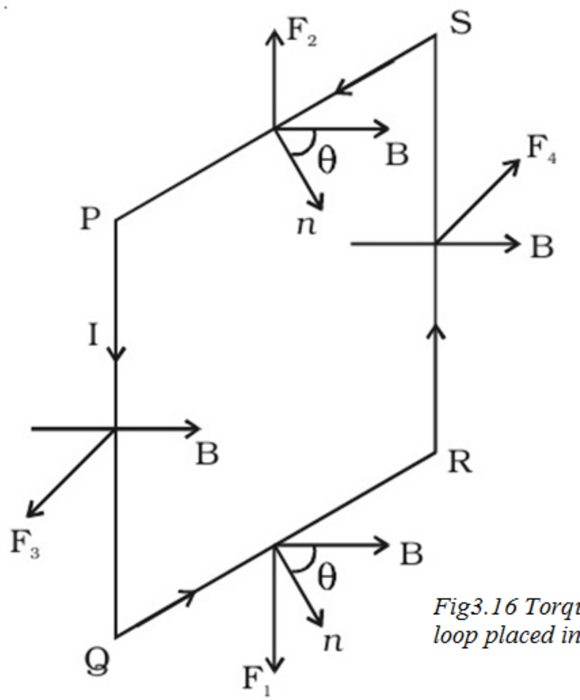


Fig3.16 Torque on a current loop placed in a magnetic field

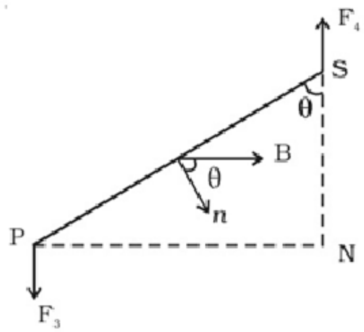


Fig3.17

F_3 acts perpendicular to the plane of the paper and outwards.

Force on the arm RS, $\vec{F_4} = I\,\vec{RS} \times \vec{B}$

Since the angle between $\vec{RS}$ and $\vec{B}$ is 90°,

Magnitude of the force $F_4 = BI\,l \sin 90° = BI\,l$

F_4 acts perpendicular to the plane of the paper and inwards.

The forces F_3 and F_4 are equal in magnitude, opposite in direction and have different lines of action. So, they constitute a couple.

Hence, Torque = $BI\,l \times PN = BI\,l \times PS \times \sin\theta$ (Fig 3.17)

$= BI\,l \times b \sin\theta = BIA \sin\theta$

If the coil contains N turns, $\tau = NBIA \sin\theta$

In vector form $\vec{\tau} = NBIA \sin\theta\,\hat{n}$, where $\hat{n}$ is unit vector normal to the plane of the loop.

$\vec{\tau} = N I(\vec{A} \times \vec{B})$ or $N(\vec{M} \times \vec{B})$, Since $\vec{M} = I\vec{A}$ is the magnetic dipole moment.

So, the torque is maximum when the coil is parallel to the magnetic field and zero when the coil is perpendicular to the magnetic field.

3.6.1 Moving coil galvanometer

Moving coil galvanometer is a device used for measuring the current in a circuit.

Principle

Moving coil galvanometer works on the principle that a current carrying coil placed in a magnetic field experiences a torque.

Construction

It consists of a rectangular coil of a large number of turns of thin insulated copper wire wound over a light metallic frame (Fig 3.18). The coil is suspended between the pole pieces of a horse-shoe magnet by a fine phosphor – bronze strip from a movable torsion head. The lower end of the coil is connected to a hair spring (HS) of phosphor bronze having only a few turns. The other end of the spring is connected to a binding screw. A soft iron cylinder is placed symmetrically inside the coil. The hemispherical magnetic poles produce a radial magnetic field in which the plane of the coil is parallel to the magnetic field in all its positions (Fig 3.19).

A small plane mirror (m) attached to the suspension wire is used along with a lamp and scale arrangement to measure the deflection of the coil.

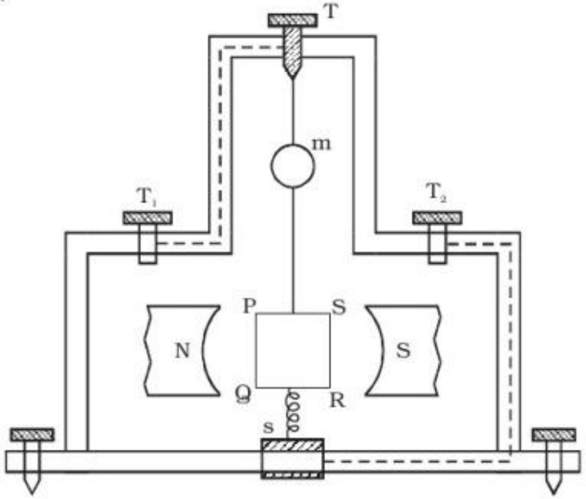


Fig 3.18 Moving coil galvanometer

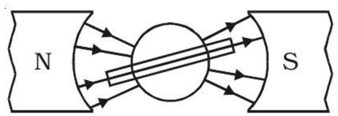


Fig 3.19 Radial magnetic field

Theory

Let PQRS be a single turn of the coil (Fig 3.20). A current I flows through the coil. In a radial magnetic field, the plane of the coil is always parallel to the magnetic field. Hence the sides QR and SP are always parallel to the field. So, they do not experience any force. The sides PQ and RS are always perpendicular to the field.

PQ = RS = l, length of the coil and PS = QR = b, breadth of the coil

Force on PQ, $F = BI(PQ) = BI\,l$ According to Fleming's left hand rule, this force is normal to the plane of the coil and acts outwards.

Force on RS, $F = BI(RS) = BI\,l$

This force is normal to the plane of the coil and acts inwards. These two equal, oppositely directed parallel forces having different lines of action constitute a couple and deflect the coil. If there are n turns in the coil,

moment of the deflecting couple = $n\,BI\,l \times b$ (Fig 3.20)

$= nBIA$

When the coil deflects, the suspension wire is twisted. On account of elasticity, a restoring couple is set up in the wire. This couple is proportional to the twist. If θ is the angular twist, then,

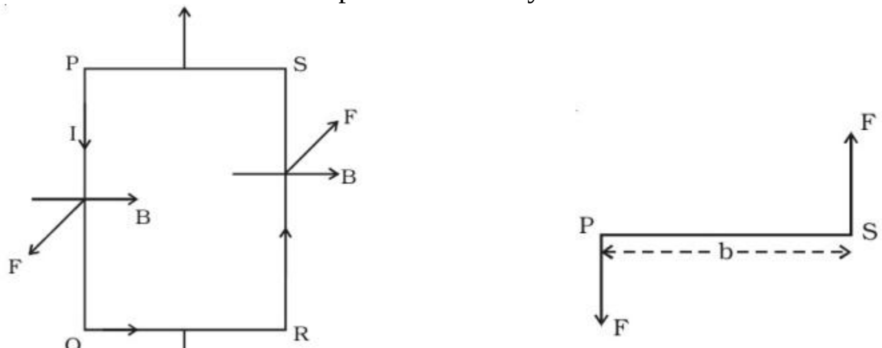


Fig 3.20 *Torque on the coil*

moment of the restoring couple = C θ

where C is the restoring couple per unit twist

At equilibrium, deflecting couple = restoring couple

nBIA = C θ

$\therefore$ I = $\frac{C}{nBA}\theta$

I = K θ where K = $\frac{C}{nBA}$ is the galvanometer constant.

i.e I α θ . Since the deflection is directly proportional to the current flowing through the coil, the scale is linear and is calibrated to give directly the value of the current.

3.6.2 Pointer type moving coil galvanometer

The suspended coil galvanometers are very sensitive. They can measure current of the order of 10-8 ampere. Hence these galvanometers have to be carefully handled. So, in the laboratory, for experiments like Wheatstone's bridge, where sensitivity is not required, pointer type galvanometers are used.

In this type of galvanometer, the coil is pivoted on ball bearings. A lighter aluminium pointer attached to the coil moves over a scale when current is passed. The restoring couple is provided by a spring.

3.6.3 Current sensitivity of a galvanometer.

The current sensitivity of a galvanometer is defined as the deflection produced when unit current passes through the galvanometer. A galvanometer is said to be sensitive if it produces large deflection for a small current.

In a galvanometer, I = $\frac{C}{nBA}\theta$

$\therefore$ Current sensitivity $\frac{\theta}{I} = \frac{nBA}{C}$ …(1)

The current sensitivity of a galvanometer can be increased by

1. increasing the number of turns
2. increasing the magnetic induction
3. increasing the area of the coil
4. decreasing the couple per unit twist of the suspension wire.

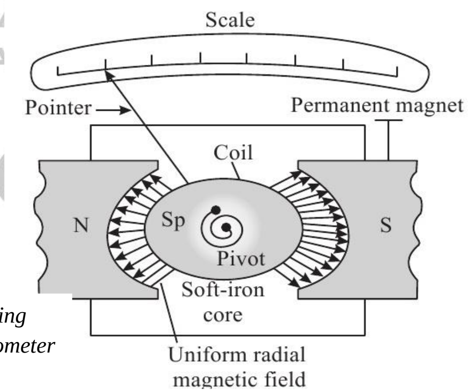


Fig3.21Moving coil Galvanometer

This explains why phosphor-bronze wire is used as the suspension wire which has small couple per unit twist.

3.6.4 Voltage sensitivity of a galvanometer

The voltage sensitivity of a galvanometer is defined as the deflection per unit voltage.

$\therefore$ Voltage sensitivity $\frac{\theta}{V} = \frac{\theta}{IG} = \frac{nBA}{CG}$ ...(2)

where G is the galvanometer resistance.

An interesting point to note is that, increasing the current sensitivity does not necessarily, increase the voltage sensitivity. When the number of turns (n) is doubled, current sensitivity is also doubled (equation 1). But increasing the number of turns correspondingly increases the resistance (G). Hence voltage sensitivity remains unchanged.

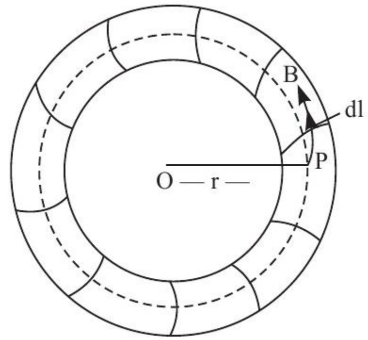
Conversion of galvanometer into an ammeter or into a voltmeter: A galvanometer is converted into an ammeter by connecting a low resistance (shunt resistance) in parallel with it, whereas it can be converted into a voltmeter by connecting a high resistance in series with it. (For detail refer 2.12)

3.7 Toroid

If the solenoid is bent in the form of a circle and its two ends are connected to each other then the device is called a toroid. It can be prepared by closely winding an insulated conducting wire around a non-conducting hollow ring. The magnetic field produced inside the toroid carrying electric current can be obtained using Ampere's circuital law.

To find a magnetic field at a point P inside a toroid which is at a distance r from its centre, consider a circle of radius r with its centre at O as an Amperean loop. By symmetry, the magnitude of the magnetic field at every point on the loop is the same and is directed towards the tangent to the circle.

Fig 3.24 Toroid



$\therefore \oint \vec{B}.\vec{dl} = \oint Bdl$ = B$\oint dl$ = B(2πr) … … … (1)

If the total number of turns is N and current is I, the total current through the said loop is NI. From Ampere's circuital law,

$\oint \vec{B}.\vec{dl} = \mu_o$ N I … … … (2)

Comparing equations (1) and (2), B (2π r) = μ_o N I

$\therefore$ B = $\frac{\mu_o N I}{2\pi r}$ = μ_o n I, (n = $\frac{N}{2\pi r}$ is the number of turns per u

In an ideal toroid, where the turns are completely circular, magnetic field at the centre and outside the toroid is zero. In practice, the coil is helical and hence a small magnetic field exists outside the toroid.

Toroid is a very important component of Tokamak used for research in nuclear fusion.

3.7 Motion of a Charged Particle in uniform Electric Field and Magnetic Field (Discussion)

(a) Motion in Electric Field

When a charged particle q is placed in a uniform electric field E, it experiences a force,

F = qE

Thus, the charged particle will be accelerated under the influence of this force. The acceleration is given by

a = $\frac{F}{m} = \frac{qE}{m}$

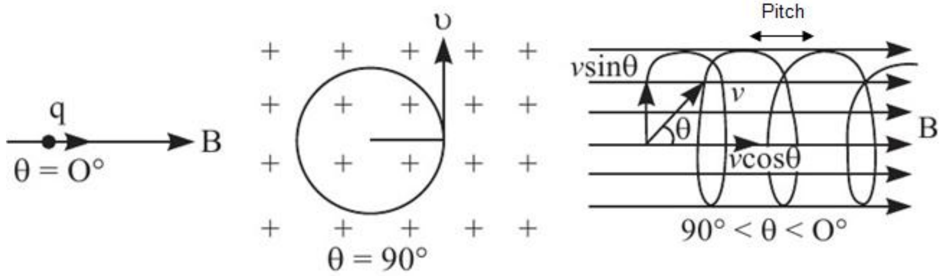
The acceleration will be in the direction of the force. If it is a positive charge, it will accelerate in the direction of the field and if it is a negative charge it will accelerate in a direction opposite that of the field. The velocity and displacement of charged particle can also be calculated by using the equations of motion:

V = u + $\left(\frac{qE}{m}\right)$ t & S = ut + $\frac{1}{2}\left(\frac{qE}{m}\right)$ t^2 where t denotes time.

(b) Motion in magnetic field

It has been discussed that the force experienced by a charged particle in a magnetic field is given by

$F = qBv\sin\theta$ Where θ is angle between the velocity and magnetic field



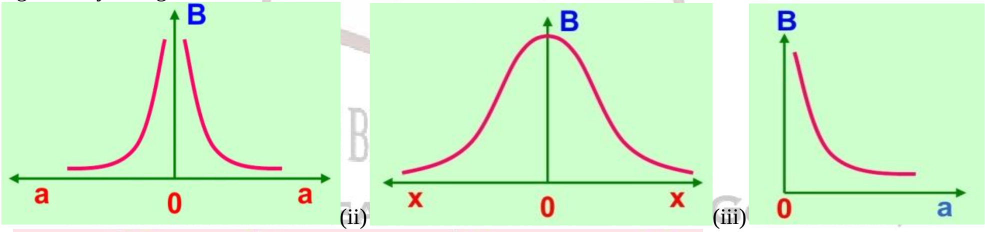
If θ = 0°, F = 0 and charged particle will move along a straight line with constant speed.

If θ = 90°, F will be maximum and its direction, according to Fleming's left hand rule, will be perpendicular to the plane of v and B and the charged particle will move along a circular path with a constant speed and frequency.

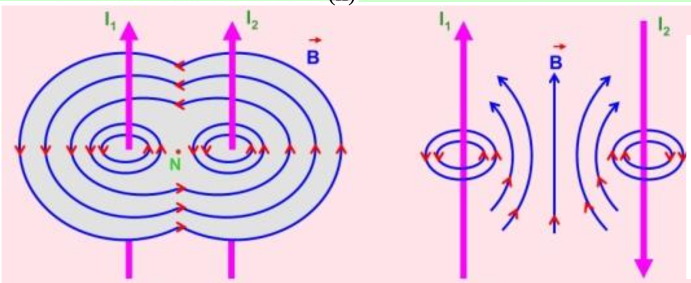
If θ ≠ 0° ≠ 90°, then the velocity of the charged particle will be $v\sin\theta$ perpendicular to the field and $v\cos\theta$ parallel to the field. $v\cos\theta$ is responsible for linear motion, $v\sin\theta$ is responsible for circular motion. The particle, therefore, moves along a helical path. When a charged particle enters in to the magnetic field with some angle θ to it, the radius of circular path followed by it is $r = mv\sin\theta/qB$, and the pitch of the helical path is $2\pi mv\cos\theta/qB$.

What we note from the above discussion is that a magnetic field does not change the speed of a moving charge, it only changes its direction of motion.

(i) (ii) (iii)



(iv)



(i)Magnetic field strength verses distance from infinitely long wire. (ii) magnetic field due to circular loop vs distance on axial line. (iii) magnetic field at centre vs radius of loop (iv) Magnetic field due to parallel current.

3.9 Current loop as a magnetic dipole

Ampere found that the distribution of magnetic lines of force around a finite current carrying solenoid is similar to that produced by a bar magnet. This is evident from the fact that a compass needle when moved

around these two bodies show similar deflections. After noting the close resemblance between these two, Ampere demonstrated that a simple current loop behaves like a bar magnet and put forward that all the magnetic phenomena is due to circulating electric current. This is Ampere's hypothesis.

The magnetic induction at a point along the axis of a circular coil carrying current is

$$B = \frac{\mu_0 nIa^2}{2(a^2+x^2)^{\frac{3}{2}}}$$

The direction of this magnetic field is along the axis and is given by right hand rule. For points which are far away from the centre of the coil, x>>a, a^2 is small and it is neglected. Hence for such points,

$$B = \frac{\mu_0 nIa^2}{2x^3}$$

If we consider a circular loop, n = 1, its area A = πa^2

$$\therefore B = \frac{\mu_0 IA}{2\pi x^3} \quad ...(1)$$

The magnetic induction at a point along the axial line of a short bar magnet is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{x^3}$$

$$B = \frac{\mu_0}{2\pi} \cdot \frac{M}{x^3} \quad ...(2)$$

Comparing equations (1) and (2), we find that

M = IA ...(3)

Hence a current loop is equivalent to a magnetic dipole of moment M = IA

The magnetic moment of a current loop is defined as the product of the current and the loop area. Its direction is perpendicular to the plane of the loop.

3.10 The magnetic dipole moment of a revolving electron

According to Neil Bohr's atom model, the negatively charged electron is revolving around a positively charged nucleus in a circular orbit of radius r. The revolving electron in a closed path constitutes an electric current. The motion of the electron in anticlockwise direction produces conventional current in clockwise direction.

Current, $i = \frac{e}{T}$ where T is the period of revolution of the electron.

If v is the orbital velocity of the electron, then

$$T = \frac{2\pi r}{v}$$

$$\therefore i = \frac{ev}{2\pi r}$$

Due to the orbital motion of the electron, there will be orbital magnetic moment μ_1

μ_1 = i A, where A is the area of the orbit

$$\mu_1 = \frac{ev}{2\pi r} \cdot \pi r^2 = \frac{evr}{2}$$

If m is the mass of the electron

$$\mu_1 = \frac{e}{2m}(mvr)$$

mvr is the angular momentum (l) of the electron about the central nucleus.

$$\mu_1 = \frac{e}{2m} l \quad \dots (1)$$

$\frac{\mu_l}{l} = \frac{e}{2m}$ is called gyromagnetic ratio and is a constant. Its value is 8.8 × 10^{10} C kg^{-1}. Bohr hypothesised that the angular momentum has only discrete set of values given by the equation.

$l = \frac{nh}{2\pi}$ ...(2)

where n is a natural number and h is the Planck's constant = 6.626×10^{-34} Js. substituting equation (2) in equation (1)

$\mu_1 = \frac{e}{2m} \cdot \frac{nh}{2\pi} = \frac{neh}{4\pi m}$

The minimum value of magnetic moment is

$(\mu_1)_{min} = \frac{eh}{4\pi m}$, n = 1

The value of $\frac{eh}{4\pi m}$ is called Bohr magneton

By substituting the values of e, h and m, the value of Bohr magneton is found to be 9.27×10^{-24} Am^2
In addition to the magnetic moment due to its orbital motion, the electron possesses magnetic moment due to its spin. Hence the resultant magnetic moment of an electron is the vector sum of its orbital magnetic moment and its spin magnetic moment.

4.1 Magnetism

The word magnetism is derived from iron ore magnetite (Fe_3O_4), which was found in the island of magnesia in Greece. It is believed that the Chinese had known the property of the magnet even in 2000 B.C. and they used magnetic compass needle for navigation in 1100 AD. But it was Gilbert who laid the foundation for magnetism and had suggested that Earth itself behaves as a giant bar magnet. The field at the surface of the Earth is approximately 10^{-4} T and the field extends upto a height of nearly five times the radius of the Earth.

4.2 Bar magnet

The iron ore magnetite which attracts small pieces of iron, cobalt, nickel etc. is a natural magnet. The natural magnets have irregular shape and they are weak. A piece of iron or steel acquires magnetic properties when it is rubbed with a magnet. Such magnets made out of iron or steel are artificial magnets. Artificial magnets can have desired shape and desired strength. If the artificial magnet is in the form of a rectangular or cylindrical bar, it is called a bar magnet.

4.2.1 Equivalence between a Bar Magnet and a solenoid

A solenoid has a large number of closely wound turns of current carrying conducting coil around a soft iron core. For clarity, a few turns separated from one another are shown. Each turn can be treated as a closed current loop possessing magnetic dipole moment. Thus each turn can be treated as a tiny magnet with north and south poles.

On looking normally at the plane of the loop, if the current appears to flow in the clockwise direction, then the side of the loop towards the eye behaves as the south pole and the other side behaves as the north pole. Similarly, if the current appears to flow in the anticlockwise direction, then the side of the loop towards the eye will behave as the north pole. This is shown in the two figures above on the right side. Thus, in the figure of the solenoid above, current in the circuit in front of the eye being in the anticlockwise direction, the side of the first turn towards the eye behaves as a North Pole, while the other side of the turn is the South Pole for that turn. For the second turn, the side towards the eye is again North Pole and so on for all the turns. Thus, magnetic dipole moment of each turn is in the same direction and hence the magnetic dipole moment of the solenoid is the vector sum of dipole moments of all the turns.

For current I through a solenoid with total number of turns N and with cross-sectional area A, the magnetic dipole moment of the solenoid is given by

$M_s = N I A$ ... ... (1)

The magnetic dipole moment of a bar magnet of pole strength, m, and length, $2l$, is given by

$M_b = 2m\, l$ ... ... (2)

By analogy between solenoid and bar magnet, the pole strength, ms, of the solenoid can be obtained using the above two equations as under.

$2m_s\, l = N I A \Rightarrow m_s = \frac{N I A}{2l} = nIA$, where, $n = \frac{N}{2l}$ = no. of turns per unit length of solenoid.

Thus Pole-strength of solenoid = Number of turns per unit length × electric current × cross-sectional area of solenoid.

The unit of pole-strength is Am (Ampere-meter)

4.2.2 Basic properties of magnets

1. When the magnet is dipped in iron filings, they cling to the ends of the magnet. The attraction is maximum at the two ends of the magnet. These ends are called poles of the magnet.
2. When a magnet is freely suspended, it always points along north-south direction. The pole pointing towards geographic north is called north pole N and the pole which points towards geographic south is called south pole S.
3. Magnetic poles always exist in pairs. (i.e) isolated magnetic pole does not exist.
4. The magnetic length of a magnet is always less than its geometric length, because the poles are situated a little inwards from the free ends of the magnet. (But for the purpose of calculation the geometric length is always taken as magnetic length.)
5. Like poles repel each other and unlike poles attract each other. North pole of a magnet when brought near north pole of another magnet, we can observe repulsion, but when the north pole of one magnet is brought near south pole of another magnet, we observe attraction.
6. The force between two magnetic poles is given by Coulomb's inverse square law.

Note : In recent days, the concept of magnetic poles has been completely changed. The origin of magnetism is traced only due to the flow of current. But this conventional idea of magnetic poles will be retained in this chapter.

Magnetic moment

Since any magnet has two poles, it is also called a magnetic dipole. The magnetic moment of a magnet is defined as the product of the pole strength and the distance between the two poles.

If m is the pole strength of each pole and $2\,l$ is the distance between the poles, the magnetic moment

$\vec{M} = m\,(2\vec{l})$

Magnetic moment is a vector quantity. It is denoted by M. Its unit is A m^2. Its direction is from south pole to north pole. When a bar magnet is cut into two equal parts:

(a) Longitudinally, then its pole strength becomes half i.e. $m' = \frac{m}{2}$ whereas magnetic length remain same and magnetic dipole moment becomes halved. $M' = \frac{M}{2}$

(b) Laterally, then its pole strength remains same but magnetic length becomes halved $2\,l' = \frac{2l}{2}$ and magnetic dipole moment becomes halved. $M' = \frac{M}{2}$

Magnetic field

Magnetic field is the space in which a magnetic pole experiences a force or it is the space around a magnet in which the influence of the magnet is felt.

Magnetic induction

Magnetic induction is the fundamental character of a magnetic field at a point.

Magnetic induction at a point in a magnetic field is the force experienced by unit north pole placed at that point. It is denoted by B. Its unit is $NA^{-1}m^{-1}$. It is a vector quantity. It is also called as magnetic flux density.

If a magnetic pole of strength m placed at a point in a magnetic field experiences a force F, the magnetic induction at that point is $\vec{B} = \frac{\vec{F}}{m}$

Magnetic lines of force

A magnetic line of force is a line along which a free isolated north pole would travel when it is placed in the magnetic field.

Properties of magnetic lines of force

1. Magnetic lines of forces are closed continuous curves, extending through the body of the magnet.
2. The direction of line of force is from north pole to south pole outside the magnet while it is from south pole to north pole inside the magnet.
3. The tangent to the magnetic line of force at any point gives the direction of magnetic field at that point. (i.e) it gives the direction of magnetic induction ($\vec{B}$) at that point.
4. They never intersect each other.
5. They crowd where the magnetic field is strong and thin out where the field is weak.

Magnetic flux and magnetic flux density

The number of magnetic lines of force passing through an area A is called magnetic flux. It is denoted by ϕ . Its unit is weber. It is a scalar quantity.

The number of magnetic lines of force crossing unit area kept normal to the direction of line of force is magnetic flux density. Its unit is Wb m^{-2} or tesla or N $A^{-1}m^{-1}$.

$\therefore$ Magnetic flux $\phi = \vec{B} \cdot \vec{A}$

Gauss's Law of Magnetism

As magnetic monopole does not exist magnetic field lines always form closed curves. As the number of field lines entering a closed su face equal the number of lines leaving it, the total flux over any closed surface is zero.

$$\oint_{\text{closed surface}} \vec{B} \cdot \overrightarrow{dA} = 0$$

where $\vec{B}$ is the magnetic field and $\overrightarrow{dA}$ is an infinitesimal area vector on the closed surface.

"The net magnetic flux passing through any closed surface is zero." This statement is generally called Gauss's law for magnetism.

Uniform and non-uniform magnetic field

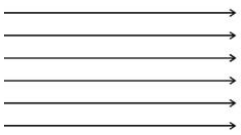


Fig 4.1 Uniform Magnetic field

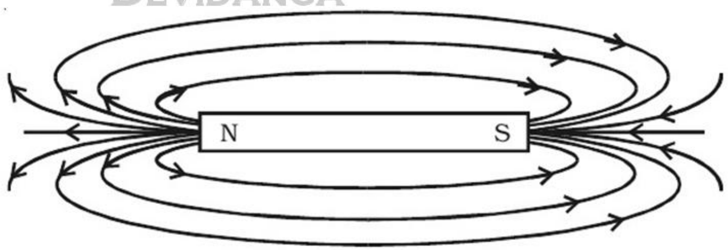


Fig 4.2 Non - Uniform Magnetic field

Magnetic field is said to be uniform if the magnetic induction has the same magnitude and the same direction at all the points in the region. It is represented by drawing parallel lines (Fig. 4.1).

An example of uniform magnetic field over a wide area is the Earth's magnetic field.

If the magnetic induction varies in magnitude and direction at different points in a region, the magnetic field is said to be non-uniform. The magnetic field due to a bar magnet is non-uniform. It is represented by convergent or divergent lines (Fig. 4.2).

4.3 Force between two magnetic poles

In 1785, Coulomb made use of his torsion balance and discovered the law governing the force between the two magnetic poles.

Coulomb's inverse square law

Coulomb's inverse square law states that the force of attraction or repulsion between the two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.

If m_1 and m_2 are the pole strengths of two magnetic poles separated by a distance of d in a medium, then

$F \propto m_1m_2$ and $F \propto \frac{1}{d^2}$

$\therefore F \propto \frac{m_1m_2}{d^2}$

$F = k\frac{m_1m_2}{d^2}$

where k is the constant of proportionality and $k = \frac{\mu}{4\pi}$ where μ is the permeability of the medium.

But $\mu = \mu_o \times \mu_r$

$\therefore \mu_r = \frac{\mu}{\mu_0}$

where μ_r - relative permeability of the medium

μ_o = permeability of free space or vacuum.

Let $m_1 = m_2 = 1$
and d = 1 m

$k = \frac{\mu_o}{4\pi}$

In free space, $\mu_o = 4\pi \times 10^{-7}$ H m^{-1}

$\therefore F = \frac{10^{-7} \times m_1 \times m_2}{d^2} = \frac{10^{-7}\times 1\times 1}{1}$ N

$F = 10^{-7}$ N

Therefore, unit pole is defined as that pole which when placed at a distance of 1 metre in free space or air from an equal and similar pole, repels it with a force of 10^{-7} N.

4.4 Magnetic induction at a point along the axial line due to a magnetic dipole (Bar magnet)

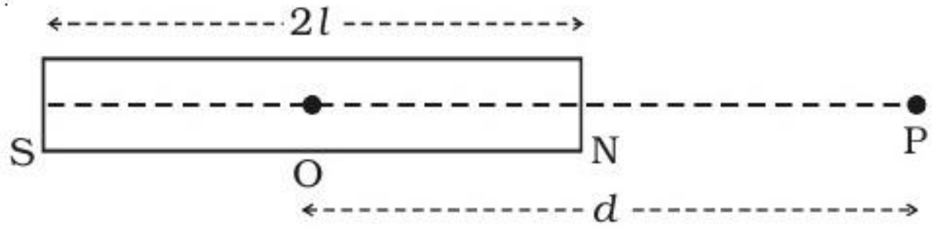


Fig. 4.3 Magnetic induction along the axial line

NS is the bar magnet of length 2 l and of pole strength m. P is a point on the axial line at a distance d from its mid point O (Fig. 4.3).

According to inverse square law, $F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{d^2}$

∴ Magnetic induction (B_1) at P due to north pole of the magnet,

$B_1 = \frac{\mu_0}{4\pi}\frac{m}{NP^2}$ along NP $(\because B = \frac{F}{M})$

$= \frac{\mu_0}{4\pi}\frac{m}{(d-l)^2}$ along NP

Magnetic induction (B_2) at P due to south pole of the magnet,

$B_2 = \frac{\mu_0}{4\pi}\frac{m}{SP^2}$ along PS

$= \frac{\mu_0}{4\pi}\frac{m}{(d+l)^2}$ along PS

∴ Magnetic induction at P due to the bar magnet,

$B = B_1 - B_2$

$B = \frac{\mu_0}{4\pi}\frac{m}{(d-l)^2} - \frac{\mu_0}{4\pi}\frac{m}{(d+l)^2}$ along NP

$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-l^2)^2}$

where $M = 2ml$ (magnetic dipole moment).

For a short bar magnet, l is very small compared to d, hence l^2 is neglected.

$\therefore B = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$

The direction of B is along the axial line away from the north pole.

4.5 Magnetic induction at a point along the equatorial line of a bar magnet

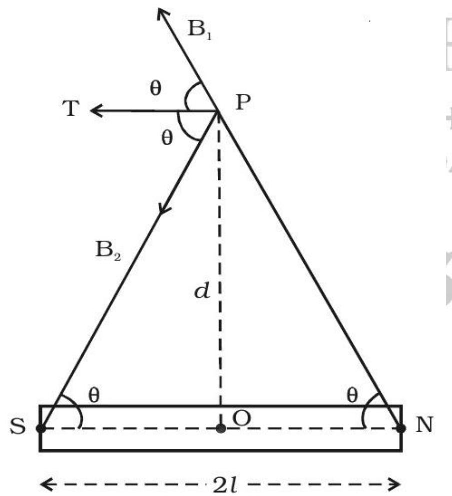


Fig. 4.4 Magnetic induction along the equatorial line

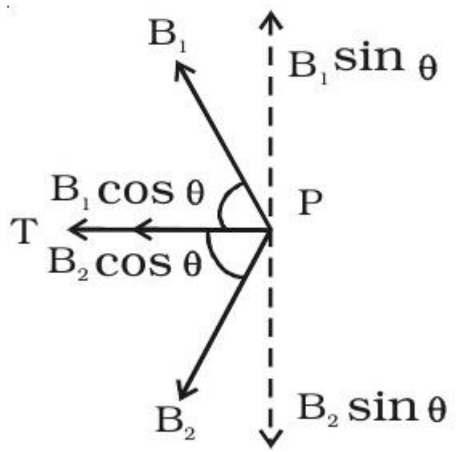


Fig. 4.5 Components of magnetic fields

NS is the bar magnet of length 2 l and pole strength m. P is a point on the equatorial line at a distance d from its mid point O (Fig. 4.4).

Magnetic induction (B_1) at P due to north pole of the magnet,

$B_1 = \frac{\mu_0}{4\pi}\frac{m}{NP^2}$ along NP

$= \frac{\mu_0}{4\pi}\frac{m}{(d^2+l^2)}$ along NP $(\because NP^2 = NO^2 + OP^2)$

Magnetic induction (B_2) at P due to south pole of the magnet,

$B_2 = \frac{\mu_0}{4\pi}\frac{m}{PS^2}$ along PS

$= \frac{\mu_0}{4\pi}\frac{m}{(d^2+l^2)}$ along PS

Resolving B_1 and B_2 into their horizontal and vertical components.

Vertical components $B_1 \sin\theta$ and $B_2 \sin\theta$ are equal and opposite and therefore cancel each other (Fig. 4.5).

The horizontal components $B_1 \cos\theta$ and $B_2 \cos\theta$ will get added along PT.

Resultant magnetic induction at P due to the bar magnet is

$B = B_1 \cos\theta + B_2 \cos\theta$. (along PT)

$B = \frac{\mu_0}{4\pi}\frac{m}{(d^2+l^2)} \cdot \frac{l}{\sqrt{(d^2+l^2)}} + \frac{\mu_0}{4\pi}\frac{m}{(d^2+l^2)} \cdot \frac{l}{\sqrt{(d^2+l^2)}}$

$B = \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}}$, (where M = 2ml)

For a short bar magnet, l^2 is neglected.

$\therefore B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$

The direction of 'B' is along PT parallel to NS.

4.6 Mapping of magnetic field due to a bar magnet

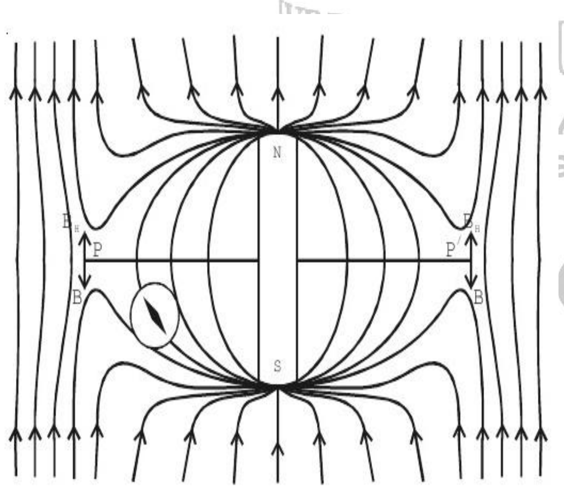


Fig. 4.6Neutral points- equatorial line

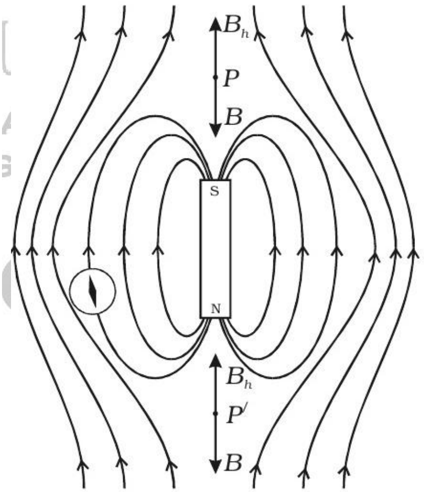


Fig. 4.7 Neutral points-axial line

(i) Magnet placed with its north pole facing geographic north

A sheet of paper is fixed on a drawing board. Using a compass needle, the magnetic meridian is drawn on it. A bar magnet is placed on the magnetic meridian such that its north pole points towards geographic north. Using a compass needle, magnetic lines of force are drawn around the magnet. (Fig. 4.6)

The magnetic lines of force is due to the combined effect of the magnetic field due to the bar magnet and Earth. It is found that when the compass is placed at points P and P′ along the equatorial line of the magnet, the compass shows no deflection. They are called "neutral points." At these points the magnetic field due to the magnet along its equatorial line (B) is exactly balanced by the horizontal component of the Earth's magnetic field. (B_h)

Hence, neutral points are defined as the points where the resultant magnetic field due to the magnet and Earth is zero.

Hence, at neutral points

$$B = B_h$$

$$\frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} = B_h$$

(ii) Magnet placed with its south pole facing geographic north

A sheet of paper is fixed on a drawing board. Using a compass needle, the magnetic meridian is drawn on it. A bar magnet is placed on a magnetic meridian such that its south pole facing geographic north. Using a compass needle, the magnetic lines of force are drawn around the magnet as shown in Fig. 4.7.

The magnetic lines of force is due to the combined effect of the magnetic field due to the bar magnet and Earth. It is found that when the compass is placed at points P and P′ along the axial line of the magnet, the compass shows no deflection. They are called neutral points. At these points the magnetic field (B) due to the magnet along its axial line is exactly balanced by the horizontal component of the Earth's magnetic field (B_h).

Hence at neutral points, $B = B_h$

$$\therefore \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-l^2)^2} = B_h$$

4.7 Torque on a bar magnet placed in a uniform magnetic field

Consider a bar magnet NS of length 2l and pole strength m placed in a uniform magnetic field of induction B at an angle θ with the direction of the field (Fig.4.8).

Due to the magnetic field B, a force mB acts on the north pole along the direction of the field and a force mB acts on the south pole along the direction opposite to the magnetic field.

These two forces are equal and opposite, hence constitute a couple.

The torque τ due to the couple is

τ = one of the forces × perpendicular distance between them

$\tau = F \times NA = mB \times NA$ ...(1)

$= mB \times 2l \sin\theta$

$\therefore \tau = MB\sin\theta$ ...(2)

Vectorially,

$$\vec{\tau} = \vec{M} \times \vec{B}$$

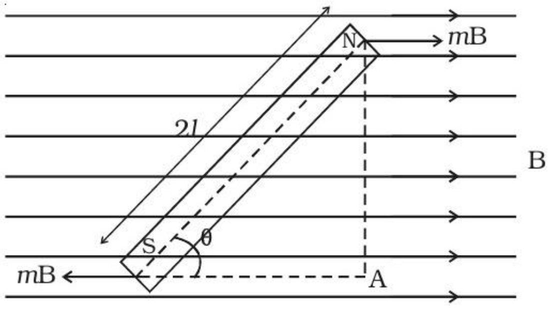


Fig. 4.8 Torque on a bar magnet

The direction of τ is perpendicular to the plane containing $\vec{M}$ and $\vec{B}$. If B = 1 and $\theta = 90°$

Then from equation (2), $\tau = M$

Hence, moment of the magnet M is equal to the torque necessary to keep the magnet at right angles to a magnetic field of unit magnetic induction.

Work done on a Magnetic Dipole (Bar Magnet) in Uniform Magnetic Field:

$dW = \tau\, d\theta$

$= M B \sin\theta\, d\theta$

$W = \int_{\theta_1}^{\theta_2} M B \sin\theta\, d\theta$

$W = M B (\cos\theta_1 - \cos\theta_2)$

If Potential Energy is arbitrarily taken zero when the dipole is at 90°, then P.E in rotating the dipole and inclining it at an angle θ is

Potential Energy = - M B cos θ

4.8 Tangent law

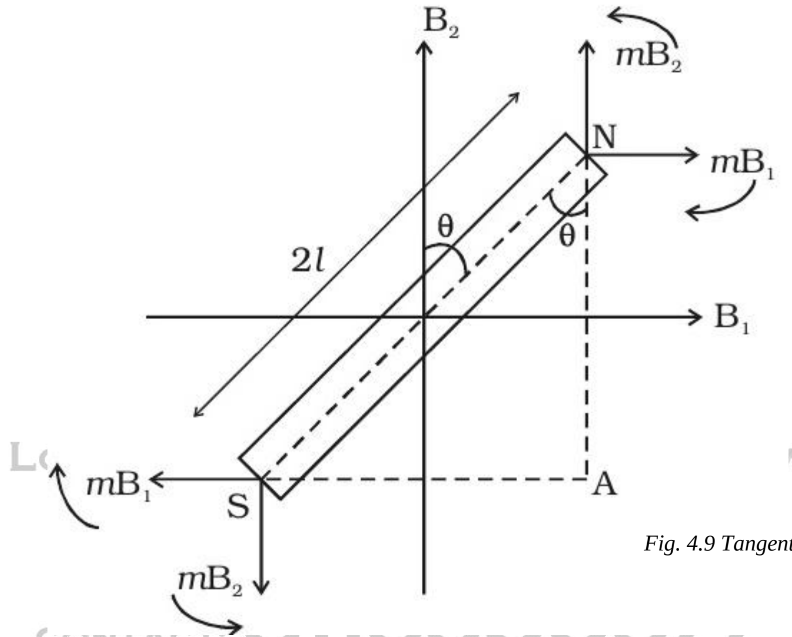


Fig. 4.9 Tangent law

A magnetic needle suspended, at a point where there are two crossed magnetic fields acting at right angles to each other, will come to rest in the direction of the resultant of the two fields.

B_1 and B_2 are two uniform magnetic fields acting at right angles to each other. A magnetic needle placed in these two fields will be subjected to two torques tending to rotate the magnet in opposite directions. The torque τ_1 due to the two equal and opposite parallel forces mB_1 and mB_1 tend to set the magnet parallel to B_1.Similarly the torque τ_2 due to the two equal and opposite parallel forces mB_2 and mB_2 tends to set the magnet parallel to B_2. In a position where the torques balance each other, the magnet comes to rest. Now the magnet makes an angle θ with B_2 as shown in the Fig. 4.9.

The deflecting torque due to the forces mB_1 and mB_1

$\tau_1 = mB_1 \times NA$

$= mB_1 \times NS \cos\theta$

$= 2\,l\, mB_1 \cos\theta$

$= MB_1 \cos\theta$

Similarly the restoring torque due to the forces mB_2 and mB_2

$\tau_2 = mB_2 \times SA$

$= mB_2 \times 2\,l \sin\theta$

$= 2lm \times B_2 \sin\theta$

$\tau_2 = MB_2 \sin\theta$

At equilibrium,

$\tau_1 = \tau_2$

$\therefore MB_1 \cos\theta = MB_2 \sin\theta$

$\therefore B_1 = B_2 \tan\theta$

This is called Tangent law

Invariably, in the applications of tangent law, the restoring magnetic field B_2 is the horizontal component of Earth's magnetic field B_h.

4.9 Magnetic properties of materials

The study of magnetic properties of materials assumes significance since these properties decide whether the material is suitable for permanent magnets or electromagnets or cores of transformers etc. Before classifying the materials depending on their magnetic behaviour, the following important terms are defined.

(i) Magnetising field or magnetic intensity

The magnetic field used to magnetise a material is called the magnetising field. It is denoted by H and its unit is A m^{-1}.

(ii) Magnetic permeability

Magnetic permeability is the ability of the material to allow the passage of magnetic lines of force through it.

Relative permeability μ_r of a material is defined as the ratio of number of magnetic lines of force per unit area B inside the material to the number of lines of force per unit area in vacuum B_0 produced by the same magnetising field.

$\therefore$ Relative permeability $\mu_r = \frac{B}{B_0}$

$$\mu_r = \frac{\mu H}{\mu_0 H} = \frac{\mu}{\mu_0}$$

(since μ r is the ratio of two identical quantities, it has no unit.)

$\therefore$ The magnetic permeability of the medium $\mu = \mu_0 \mu_r$ where μ_0 is the permeability of free space.

Magnetic permeability μ of a medium is also defined as the ratio of magnetic induction B inside the medium to the magnetising field H inside the same medium.

$\therefore \mu = \frac{B}{H}$

(iii) Intensity of magnetisation

Intensity of magnetisation represents the extent to which a material has been magnetised under the influence of magnetising field H.

Intensity of magnetisation of a magnetic material is defined as the magnetic moment per unit volume of the material.

$I = \frac{M}{V}$

Its unit is A m^{-1}.

For a specimen of length $2\,l$, area A and pole strength m,

$I = \frac{2lm}{2lA}$

$\therefore I = \frac{m}{A}$

Hence, intensity of magnetisation is also defined as the pole strength per unit area of the cross section of the material.

(iv) Magnetic induction

When a soft iron bar is placed in a uniform magnetising field H, the magnetic induction inside the specimen B is equal to the sum of the magnetic induction B_o produced in vacuum due to the magnetising field and the magnetic induction Bm due to the induced magnetisation of the specimen.

$B = B_o + B_m$

But $B_o = \mu_o H$ and $B_m = \mu_o I$

$B = \mu_o H + \mu_o I$

$\therefore B = \mu_o (H + I)$

(v) Magnetic susceptibility

Magnetic susceptibility χ_m is a property which determines how easily and how strongly a specimen can be magnetised.

Susceptibility of a magnetic material is defined as the ratio of intensity of magnetisation I induced in the material to the magnetising field H in which the material is placed.

Thus $\chi_m = \frac{I}{H}$

Since I and H are of the same dimensions, χ_m has no unit and is dimensionless.

Relation between χ_m and μ_r

$\chi_m = \frac{I}{H}$

$\therefore I = \chi_m H$

We know $B = \mu_o (H + I) = \mu_o (H + \chi_m H)$

$B = \mu_o H (1 + \chi_m)$

If μ is the permeability, we know that $B = \mu H$.

$\therefore \mu H = \mu_o H (1 + \chi_m)$

$\frac{\mu}{\mu_0} = (1 + \chi_m)$

$\therefore \quad \mu_r = 1 + \chi_m$

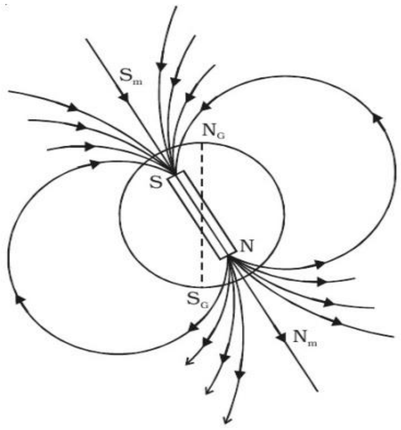


Fig 4.10 Magnetic field of earth

4.10 Earth's magnetic field and magnetic elements

A freely suspended magnetic needle at a point on Earth comes to rest approximately along the geographical north - south direction. This shows that the Earth behaves like a huge magnetic dipole with its magnetic poles near its geographical poles. Since the north pole of the magnetic needle approximately points towards geographic north (N_G) it is appropriate to call the magnetic pole near N_G as the magnetic south pole of Earth S_m. Also, the pole near S_G is the magnetic north pole of the Earth (N_m). (Fig.4.10)

Causes of the Earth's magnetism

The exact cause of the Earth's magnetism is not known even today. However, some important factors which may be the cause of Earth's magnetism are:

(i) Magnetic masses in the Earth.
(ii) Electric currents in the Earth.
(iii) Electric currents in the upper regions of the atmosphere.
(iv) Radiations from the Sun.
(v) Action of moon etc.

However, it is believed that the Earth's magnetic field is due to the molten charged metallic fluid inside the Earth's surface with a core of radius about 3500 km compared to the Earth's radius of 6400 km.

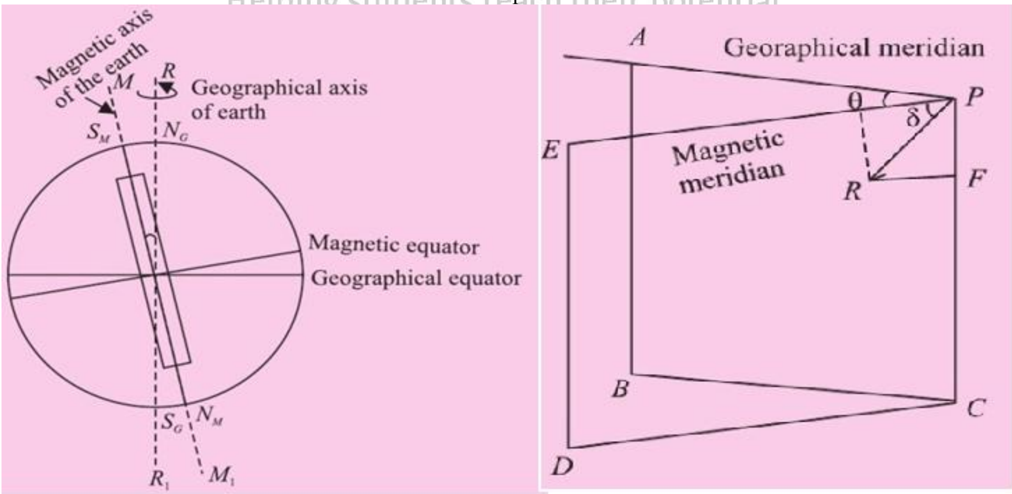


Fig 4.11 Elements of earth's magnetic field

Terrestrial Magnetism:

i) **Geographic Axis** is a straight line passing through the geographical poles of the earth. It is the axis of rotation of the earth. It is also known as polar axis.

ii) **Geographic Meridian** at any place is a vertical plane passing through the geographic north and south poles of the earth.

iii) **Geographic Equator** is a great circle on the surface of the earth, in a plane perpendicular to the geographic axis. All the points on the geographic equator are at equal distances from the geographic poles.

iv) **Magnetic Axis** is a straight line passing through the magnetic poles of the earth. It is inclined to Geographic Axis nearly at an angle of 17°.

v) **Magnetic Meridian** at any place is a vertical plane passing through the magnetic north and south poles of the earth.

vi) **Magnetic Equator** is a great circle on the surface of the earth, in a plane perpendicular to the magnetic axis. All the points on the magnetic equator are at equal distances from the magnetic poles

The Earth's magnetic field at any point on the Earth can be completely defined in terms of certain quantities called magnetic elements of the Earth, namely

(i) Declination or the magnetic variation θ.

(ii) Dip or inclination δ and
(iii) The horizontal component of the Earth's magnetic field B_h

Declination (θ):

The angle between the magnetic meridian and the geographic meridian at a place is Declination at that place. It varies from place to place.

Dip or Inclination (δ):

If you suspend a magnetic needle freely at a place, you will observe that the needle does not rest in the horizontal plane. It will point in the direction of the resultant intensity of earth's field.

The angle which the earth's magnetic field makes with the horizontal direction in the magnetic meridian is called the Dip or Inclination.

The angle between the horizontal component of earth's magnetic field and the earth's resultant magnetic field at a place is Dip or Inclination at that place. It is zero at the equator and 90°at the poles.

Horizontal component

Fig.4.11 shows that PR is the resultant magnetic field at the point P. PH represents the horizontal component and PF the vertical component of the earth's magnetic field in magnitude and direction. Let the magnetic field at the point P be B. The horizontal component

$B_H = B\cos\delta$ …..(1)

and the vertical component

$B_V = B\sin\delta$ …...(2)

By squarring and adding Eqns. (1) and (2), we get

$B_H{}^2 + B_V{}^2 = B^2\cos^2\delta + B^2\sin^2\delta = B^2$ …..(3)

On dividing Eqn. (2) by Eqn. (1), we have

$\frac{B_V}{B_H} = \tan\delta$ …..(4)

4.11Classification of magnetic materials

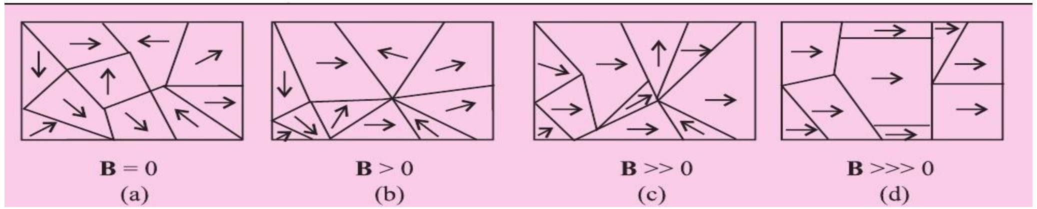


Fig 4.12 Domains in ferromagnetic substance

Based on the behaviour of materials in magnetic field, we can divide them broadly into three categories : (i) Diamagnetic materials are feebly repelled by a magnet. (ii) Paramagnetic materials are feebly attracted by a magnet. (iii) Ferromagnetic materials are very strongly attracted by a magnet. Substances like iron, nickel and cobalt are ferromagnetic. Let us study **ferromagnetic** behaviour of materials in some details.

Ferromagnetic materials, when placed even in weak magnetic field, become magnets, because their atoms act as permanent magnetic dipoles. The atomic dipoles tend to align parallel to each other in an external field. These dipoles are not independent of each other. Any dipole strongly feels the presence of a neighbouring dipole. A correct explanation of this interaction can be given only on the basis of quantum mechanics. However, we can qualitatively understand the ferromagnetic character along the following lines.

A ferromagnetic substance contains small regions called domains. All magnetic dipoles in a domain are fully aligned. The magnetization of domains is maximum. But the domains are randomly oriented. As a result, the total magnetic moment of the sample is zero. When we apply an external magnetic field, the domains slightly rotate and align themselves in the direction of the field giving rise to resultant magnetic moment. The process can be easily understood with the help of a simple diagram shown in Fig.4.12.

Fig. 4.12 (a) shows ten domains. For simplicity we take a two dimensional example. All the domains are so directed that the total magnetization of the sample is zero. Fig. 4.12 (b) shows the state after the application of an external magnetic field. The boundaries of the domains (Domain Walls) reorganise in such a way that the size of the domain having magnetic moment in the direction of the field becomes larger at the cost of others. On increasing the strength of external field, the size of favorable domains increases, and the orientation of the domain changes slightly resulting in greater magnetization (Fig. 4.12 (c)). Under the action of very strong applied field, almost the entire volume behaves like a single domain giving rise to saturated magnetization. When the external field is removed, the sample retains net magnetization. The domain in ferromagnetic samples can be easily seen with the help of high power microscope.

When the temperature of a ferromagnetic substance is raised beyond a certain critical value, the substance becomes paramagnetic. This critical temperature is known as Curie temperature Tc.

Curie's law

Some paramagnetic materials are aluminium, sodium, calcium, oxygen (at STP) and copper chloride. Experimentally, one finds that the magnetisation of a paramagnetic material is inversely proportional to the absolute temperature T,

$$I = C\frac{B_0}{T}$$

or $$\chi_m = C\frac{\mu_0}{T}$$

This is known as Curie's law, after its discoverer Pieree Curie (1859- 1906). The constant C is called Curie's constant. Thus, for a paramagnetic material both χ and μr depend not only on the material, but also (in a simple fashion) on the sample temperature. As the field is increased or the temperature is lowered, the magnetisation increases until it reaches the saturation value Ms, at which point all the dipoles are perfectly aligned with the field. Beyond this, Curie's law is no longer valid.

Properties of diamagnetic substances

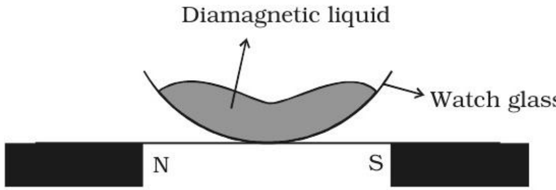


Fig. 4.13 Diamagnetic liquid

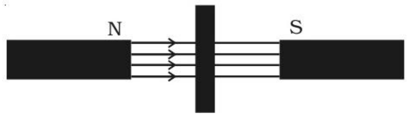


Fig. 4.14 Diamagnetic material perpendicular to field

Diamagnetic substances are those in which the net magnetic moment of atoms is zero.

1. The susceptibility has a low negative value. (For example, for bismuth $\chi_m = -0.00017$).
2. Susceptibility is independent of temperature.
3. The relative permeability is slightly less than one.
4. When placed in a non uniform magnetic field they have a tendency to move away from the field. (i.e) from the stronger part to the weaker part of the field. They get magnetised in a direction opposite to the field as shown in the Fig. 4.13.
5. When suspended freely in a uniform magnetic field, they set themselves perpendicular to the direction of the magnetic field (Fig. 4.14). Examples : Bi, Sb, Cu, Au, Hg, H_2O, H_2 etc.

Properties of paramagnetic substances

Paramagnetic substances are those in which each atom or molecule has a net non-zero magnetic moment of its own.

1. Susceptibility has a low positive value. (For example : χ_m for aluminium is +0.00002).
2. Susceptibiltity is inversely proportional to absolute temperature (i.e) $\chi_m \propto \frac{1}{T}$. As the temperature increases susceptibility decreases.

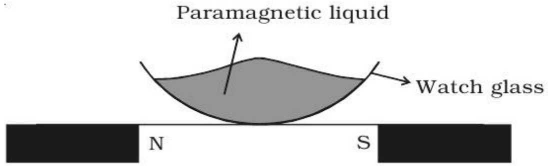


Fig. 4.15 Paramagnetic liquid

Fig. 4.16 Paramagnetic material parallel to field

3. The relative permeability is greater than one.
4. When placed in a non uniform magnetic field, they have a tendency to move from weaker part to the stronger part of the field. They get magnetised in the direction of the field as shown in Fig. 4.15.
5. When suspended freely in a uniform magnetic field, they set themselves parallel to the direction of magnetic field (Fig. 4.16). Examples : Al, Pt, Cr, O_2, Mn, $CuSO_4$ etc.

Properties of ferromagnetic substances

Ferromagnetic substances are those in which each atom or molecule has a strong spontaneous net magnetic moment. These substances exhibit strong paramagnetic properties.

1. The susceptibility and relative permeability are very large. (For example : μ_r for iron = 200,000)
2. Susceptibility is inversely proportional to the absolute temperature. (i.e) $\chi_m \propto \frac{1}{T}$. As the temperature increases the value of susceptibility decreases. At a particular temperature, ferro magnetics become para magnetics. This transition temperature is called curie temperature. For example curie temperature of iron is about 1000 K.
3. When suspended freely in uniform magnetic field, they set themselves parallel to the direction of magnetic field.
4. When placed in a non uniform magnetic field, they have a tendency to move from the weaker part to the stronger part of the field. They get strongly magnetised in the direction of the field. Examples : Fe, Ni, Co and a number of their alloys.

4.12 Hysteresis

Consider an iron bar being magnetised slowly by a magnetising field H whose strength can be changed. It is found that the magnetic induction B inside the material increases with the strength of the magnetising field and then attains a saturated level. This is depicted by the path OP in the Fig4.17.

If the magnetising field is now decreased slowly, then magnetic induction also decreases but it does not follow the path PO. Instead, when H = 0, B has nonzero value equal to OQ. This implies that some magnetism is left in the specimen. The value of magnetic induction of a substance, when the magnetising field is reduced to zero, is called remanance or residual magnetic induction of the material. OQ represents the residual magnetism of

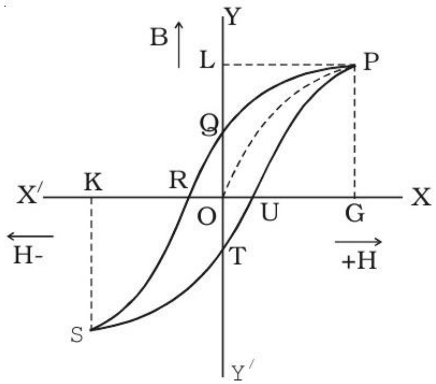


Fig. 4.17 Hysterisis loop

the material. Now, if we apply the magnetising field in the reverse direction, the magnetic induction decreases along QR till it becomes zero at R. Thus to reduce the residual magnetism (remanent magnetism) to zero, we have to apply a magnetising field OR in the opposite direction.

The value of the magnetising field H which has to be applied to the magnetic material in the reverse direction so as to reduce its residual magnetism to zero is called its coercivity.

When the strength of the magnetising field H is further increased in the reverse direction, the magnetic induction increases along RS till it acquires saturation at a point S (points P and S are symmetrical). If we now again change the direction of the field, the magnetic induction follows the path STUP. This closed curve PQRSTUP is called the 'hysteresis loop' and it represents a cycle of magnetisation. The word 'hysteresis' literally means lagging behind. We have seen that magnetic induction B lags behind the magnetising field H in a cycle of magnetisation. This phenomenon of lagging of magnetic induction behind the magnetising field is called hysteresis.

Hysteresis loss

In the process of magnetisation of a ferromagnetic substance through a cycle, there is expenditure of energy. The energy spent in magnetising a specimen is not recoverable and there occurs a loss of energy in the form of heat. This is so because, during a cycle of magnetisation, the molecular magnets in the specimen are oriented and reoriented a number of times. This molecular motion results in the production of heat. It has been found that loss of heat energy per unit volume of the specimen in each cycle of magnetisation is equal to the area of the hysteresis loop.

The shape and size of the hysteresis loop is characteristic of each material because of the differences in their retentivity, coercivity, permeability, susceptibility and energy losses etc. By studying hysteresis loops of various materials, one can select suitable materials for different purposes.

4.12.1 Uses of ferromagnetic materials

(i) Permanent magnets

The ideal material for making permanent magnets should possess high retentivity (residual magnetism) and high coercivity so that the magnetisation lasts for a longer time. Examples of such substances are steel and alnico (an alloy of Al, Ni and Co).

(ii) Electromagnets

Material used for making an electromagnet has to undergo cyclic changes. Therefore, the ideal material for making an electromagnet has to be one which has the least hysteresis loss. Moreover, the material should attain high values of magnetic induction B at low values of magnetising field H. Soft iron is preferred for making electromagnets as it has a thin hysteresis loop (Fig. 4.18) [small area, therefore less hysteresis loss] and low retentivity. It attains high values of B at low values of magnetising field H.

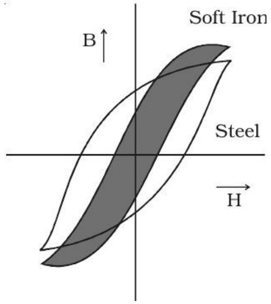


Fig. 4.18 Hysteresis loop for steel and soft iron

(iii) Core of the transformer

A material used for making transformer core and choke is subjected to cyclic changes very rapidly. Also, the material must have a large value of magnetic induction B. Therefore, soft iron that has thin and tall hysteresis loop is preferred. Some alloys with low hysteresis loss are: radio-metals, pern-alloy and mumetal.

(iv) Magnetic tapes and memory store

Magnetisation of a magnet depends not only on the magnetising field but also on the cycle of magnetisation it has undergone. Thus, the value of magnetisation of the specimen is a record of the cycles of magnetisation it has undergone. Therefore, such a system can act as a device for storing memory. Ferro magnetic materials are

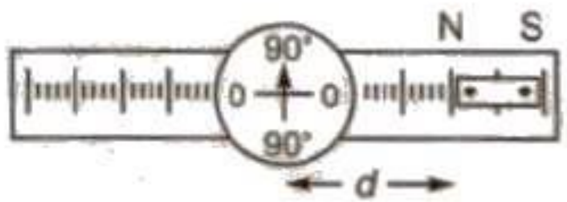
used for coating magnetic tapes in a cassette player and for building a memory store in a modern computer. Examples : Ferrites (Fe, Fe_2O, $MnFe_2O_4$ etc.)

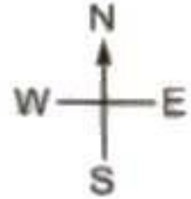
4.13 Deflection Magnetometer

It is a device used to determine M and H. Its working is based on tangent law. Deflection magnetometer can be used into two settings

(i) Tangent A setting

In this setting the arms of the magnetometer are along east-west and magnet is parallel to the arms.





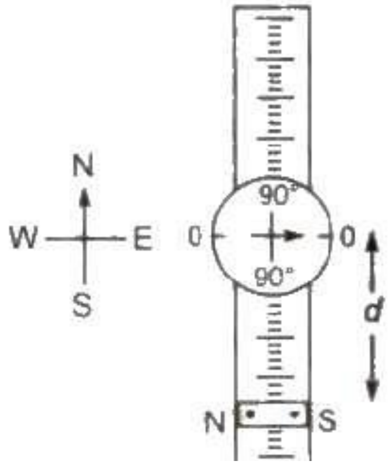
In equilibrium $B = H \tan\theta$

$$\frac{\mu_0}{4\pi}\frac{2M}{d^3} = H \tan\theta$$

(ii) Tangent B setting

In this setting the arms of the magnetometer are along north -south and magnet is perpendicular to these arm in equilibrium

$$\frac{\mu_0}{4\pi}\frac{M}{d^3} = H \tan\theta$$



In above setting the experiment can be performed in two ways.

(a) Deflection method: In this method one magnet is used at a time and deflection in galvanometer is observed. Ratio of magnetic dipole moments of the magnets

$$M_1 / M_2 = \tan\theta_1 / \tan\theta_2$$

where θ_1 and θ_2 are mean values of deflection for two magnets.

(b) Null method: In this method both magnets are used at a time and no deflection condition is obtained. If Magnets are at distance d_1 and d_2 then

$$M_1 / M_2 = (d_1 / d_2)^3$$