Electrostatics

1.1 Electrostatics

In 600 B.C., Thales, a Greek Philosopher observed that, when a piece of amber is rubbed with fur, it acquires the property of attracting light objects like bits of paper. In the 17th century, William Gilbert discovered that, glass, ebonite etc, also exhibit this property, when rubbed with suitable materials.

The substances which acquire charges on rubbing are said to be 'electrified' or charged. These terms are derived from the Greek word elektron, meaning amber. Electrostatics is the branch of Physics, which deals with static electric charges or charges at rest. The charges in a electrostatic field are analogous to masses in a gravitational field. These charges have forces acting on them and hence possess potential energy.

1.1.1 Two kinds of charges

(i) If a glass rod is rubbed with a silk cloth, it acquires positive charge while the silk cloth acquires an equal amount of negative charge.

(ii) If an ebonite rod is rubbed with fur, it becomes negatively charged, while the fur acquires equal amount of positive charge. This classification of positive and negative charges was termed by American scientist, Benjamin Franklin.

Thus, charging a rod by rubbing does not create electricity, but simply transfers or redistributes the charges in a material.

1.1.2 Like charges repel and unlike charges attract each other

A charged glass rod is suspended by a silk thread, such that it swings horizontally. Now another charged glass rod is brought near the end of the suspended glass rod. It is found that the ends of the two rods repel each other. However, if a charged ebonite rod is brought near the end of the suspended rod, the two rods attract each other. This experiment shows that like charges repel and unlike charges attract each other

1.1.3 Conductors and Insulators

According to the electrostatic behaviour, materials are divided into two categories: conductors and insulators (dielectrics). Bodies which allow the charges to pass through are called conductors. e.g. metals, Earth etc. Bodies which do not allow the charges to pass through are called insulators. e.g. glass, mica, ebonite, plastic etc.

1.1.4 Basic properties of electric charge

(i) Quantisation of electric charge: The fundamental unit of electric charge (e) is the charge carried by the electron and its unit is coulomb. e has the magnitude 1.6×10^{-19}C.

In nature, the electric charge of any system is always an integral multiple of the least amount of charge. It means that the quantity can take only one of the discrete set of values. The charge, q = ne where n is an integer.

(ii) Conservation of electric charge: Electric charges can neither be created nor destroyed. According to the law of conservation of electric charge, the total charge in an isolated system always remains constant. But the charges can be transferred from one part of the system to another, such that the total charge always remains conserved. For example, Uranium ($_{92}U^{238}$) can decay by emitting an alpha particle ($_2He^4$ nucleus) and transforming to thorium ($_{90}Th^{234}$).

$$_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_2He^4$$

Total charge before decay = +92e, total charge after decay = 90e + 2e. Hence, the total charge is conserved. i.e. it remains constant.

(iii) Additive nature of charge: The total electric charge of a system is equal to the algebraic sum of electric charges located in the system. For example, if two charged bodies of charges +2q, −5q are brought in contact, the total charge of the system is –3q.

1.1.5 Coulomb's law

The force between two charged bodies was studied by Coulomb in 1785. Coulomb's law states that the force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

The direction of forces is along the line joining the two point charges. Let q_1 and q_2 be two point charges placed in air or vacuum at a distance r apart (fig1.3a). Then, according to Coulomb's law,

$$F \propto \frac{q_1 q_2}{r^2} \quad \text{or } F = k\,\frac{q_1 q_2}{r^2}$$

Where k is a constant of proportionality. In air or vacuum, $k = \frac{1}{4\pi\varepsilon_0}$, where ε_0 is the permittivity of free space (i.e., vacuum) and the value of ε_0 is 8.854×10^{-12} C² N⁻¹m⁻².

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \quad \ldots(1)$$

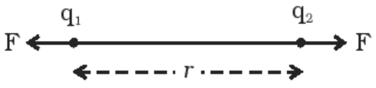


Fig 1.3a Coulomb forces

$$\text{And } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

In the above equation, if $q_1 = q_2$ = 1C and r = 1m then,

$$F = (9 \times 10^9)\,\frac{1\times1}{1^2} = 9 \times 10^9 \text{ N}$$

One Coulomb is defined as the quantity of charge, which when placed at a distance of 1 metre in air or vacuum from an equal and similar charge, experiences a repulsive force of 9×10^9 N.

If the charges are situated in a medium of permittivity ε, then the magnitude of the force between them will be,

$$F_m = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \quad \ldots(2)$$

Dividing equation (1) by (2)

$$\frac{F}{F_m} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$$

The ratio $\frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$, is called the relative permittivity or dielectric constant of the medium.

Since $F_m = \frac{F}{\varepsilon_r}$, the force between two point charges depends on the nature of the medium in which the two charges are situated. [NOTE: $\frac{\varepsilon}{\varepsilon_0} = \varepsilon_r \therefore \varepsilon = \varepsilon_0\varepsilon_r$ .The value of ε_r for air or vacuum is 1. $\therefore \varepsilon_{vacuum} = \varepsilon_0$]

Coulomb's law – vector form

If $\vec{F}_{12}$ is the force exerted on charge q_1 by charge q_2, and $\hat{r}_{12}$ is the unit vector from q_2 to q_1 .

$$\vec{F}_{12} = k\,\frac{q_1 q_2}{|\vec{r}_{12}|^2}\,\hat{r}_{12}$$

$$= k\,\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2}\,\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}$$

$$\hat{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}$$

$$\vec{F}_{12} = k\,\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) = k\,\frac{q_1 q_2}{r^2}\,\hat{r}_{12}$$

If $\vec{F}_{21}$ is the force exerted on q_2 due to q_1,

$$\vec{F}_{21} = k\,\frac{q_1 q_2}{{r_{21}}^2}\,\hat{r}_{21}$$

where $\hat{r}_{21}$ is the unit vector from q_2 to q_1.

[Both $\hat{r}_{21}$ and $\hat{r}_{12}$ have the same magnitude (r), and are oppositely directed]

$$\therefore \vec{F}_{21} = k\,\frac{q_1 q_2}{r^2}(-\hat{r}_{21})$$

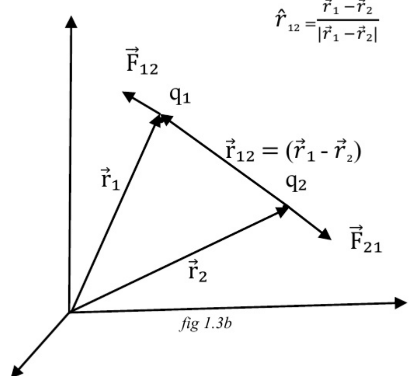


fig 1.3b

or $\vec{F}_{21} = -k \frac{q_1 q_2}{r^2}\hat{r}_{12}$

or $\vec{F}_{21} = -\vec{F}_{12}$

So, the forces exerted by charges on each other are equal in magnitude and opposite in direction.

1.1.6 Principle of Superposition

The principle of superposition is to calculate the electric force experienced by a charge q_1 due to other charges $q_2, q_3 \ldots\ldots q_n$. The total force on a given charge is the vector sum of the forces exerted on it due to all other charges. The force on q_1 due to q_2

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

Similarly, force on q_1 due to q_3

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13}$$

The total force $\vec{F}_1$ on the charge q_1 by all other charges is,

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} \ldots\ldots\ldots + \vec{F}_{1n}$$

Therefore, $$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} + \cdots + \frac{q_1 q_n}{r_{1n}^2}\hat{r}_{1n}\right]$$

[NOTE: (i) In new theories of proton and neutrons, a required constituent particles called Quarks which carry charges ± (1/3)e or ± (2/3)e. But because free quarks do not exist and their sum is always an integral number, it does not violet the quantization rules.
(ii) Charge and Mass relation: Charge cannot exist without matter. One carrier of charge is electron which has mass as well. Hence if there is charge transfer, mass is also transferred. Logically, negatively charged body is heavier then positively charged body.
(iii) In electrostatic cgs system, the unit of charge is known as electrostatic unit of charge (esu) or statcoulomb (stat C) $1C = 3 \times 10^9$ esu]

1.2 Electric Field

Electric field due to a charge is the space around the test charge in which it experiences a force. The presence of an electric field around a charge cannot be detected unless another charge is brought towards it.
When a test charge q_0 is placed near a charge q, which is the source of electric field, an electrostatic force F will act on the test charge.

Electric Field Intensity (E)

Electric field at a point is measured in terms of electric field intensity. Electric field intensity at a point, in an electric field is defined as the force experienced by a unit positive charge kept at that point.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

It is a vector quantity. The unit of electric field intensity is N C^{-1}.

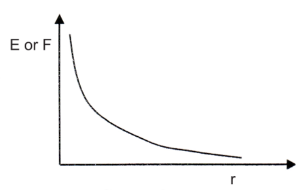
The electric field intensity is also referred as electric field strength or simply electric field. So, the force exerted by an electric field on a charge is $\vec{F} = q_0\vec{E}$.

There is difficulty in defining the electric field by the above equation. The test charge q_0 may disturb the charge distribution of the source charge and hence change the electric field $\vec{E}$ which we want to measure. The test charge must be small enough so that it does not change value of $\vec{E}$. So

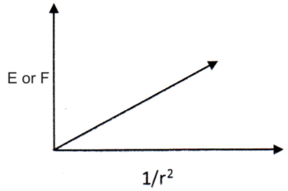
$$\vec{E} = \lim_{\Delta q \to 0}\frac{\vec{F}}{\Delta q};$$ where Δq is positive test charge.

1.2.1 Electric field due to a point charge

Fig 1.4 Electric field due to a point charge



(i) Graph for electric force (F) or electric field (E) verses distance r



(ii) Graph for E or E verses 1/r²

Let q be the point charge placed at O in air (fig1.4). A test charge q_0 is placed at P at a distance r from q. According to Coulomb's law, the force acting on q_0 due to q is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r^2}$$

The electric field at a point P is, by definition, the force per unit test charge.

$$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

The direction of E is along the line joining O and P, pointing away from q, if q is positive and towards q, if q is negative.

In vector notation $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$, where $\hat{r}$ is a unit vector pointing away from q.

1.2.2 Electric field due to system of charges

If there are a number of stationary charges, the net electric field (intensity) at a point is the vector sum of the individual electric fields due to each charge.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 \ldots\ldots\ldots + \vec{E}_n$$

Therefore,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \cdots + \frac{q_n}{r_n^2}\hat{r}_n\right]$$

1.2.3 Electric lines of force

The concept of field lines was introduced by Michael Faraday as an aid in visualizing electric and magnetic fields. Electric line of force is an imaginary straight or curved path along which a unit positive charge tends to move in an electric field.

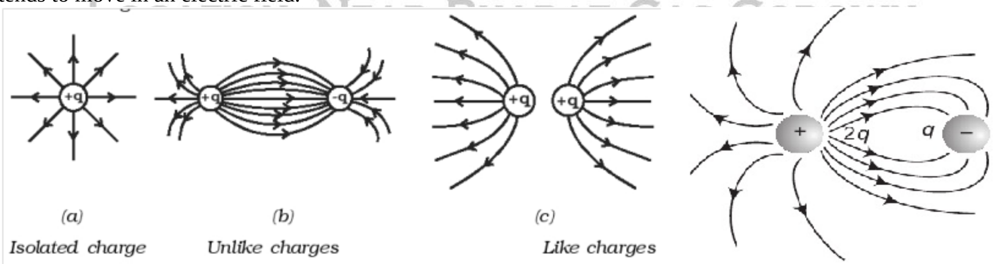


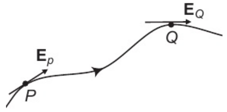
Fig1.5 Lines of Forces

Properties of lines of forces:

a. Lines of force start from positive charge and terminate at negative charge.
b. Lines of force never intersect.
c. The tangent to a line of force at any point gives the direction of the electric field (E) at that point.

d. The number of lines per unit area, through a plane at right angles to the lines, is proportional to the magnitude of E. This means that, where the lines of force are close together, E is large and where they are far apart, E is small.

e. Each unit positive charge gives rise to $\frac{1}{\varepsilon_0}$ lines of force in free space. Hence number of lines of force originating from a point charge q is N = $\frac{q}{\varepsilon_0}$ in free space.



1.2.4 Electric dipole and electric dipole moment

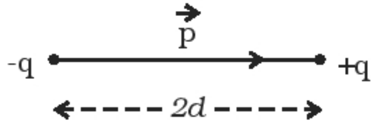


Fig1.6 Electric dipole

Two equal and opposite charges separated by a very small distance constitute an electric dipole. Water, ammonia, carbon−dioxide and chloroform molecules are some examples of permanent electric dipoles.

These molecules behave like electric dipole, because the centres of positive and negative charge do not coincide and are separated by a small distance.

Two point charges +q and –q are kept at a distance 2d apart (Fig.1.6). The magnitude of the dipole moment is given by the product of the magnitude of the one of the charges and the distance between them.

∴ Electric dipole moment, p = q.2d or 2qd.

It is a vector quantity and acts from –q to +q. The unit of dipole moment is C m.

1.2.5 Electric field due to an electric dipole at a point on its axial line.

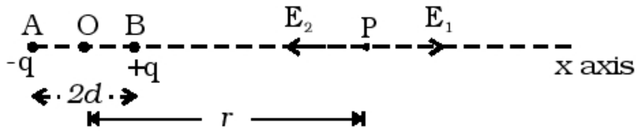


Fig 1.7 Electric field at a point on the axial line

AB is an electric dipole of two point charges –q and +q separated by a small distance 2d (fig1.7). P is a point along the axial line of the dipole at a distance r from the midpoint O of the electric dipole

The electric field at the point P due to +q placed at B is,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-d)^2} \text{ (along BP)}$$

The electric field at the point P due to –q placed at A is,

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+d)^2} \text{ (along PA)}$$

E_1 and E_2 act in opposite directions. Therefore, the magnitude of resultant electric field (E) acts in the direction of the vector with a greater magnitude. The resultant electric field at P is,

$$\vec{E} = \vec{E}_1 + (-\vec{E}_2) = \left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-d)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+d)^2}\right] \text{ along BP}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-d)^2} - \frac{1}{(r+d)^2}\right] \text{ along BP}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4rd}{(r^2-d^2)^2}\right] \text{ along BP}$$

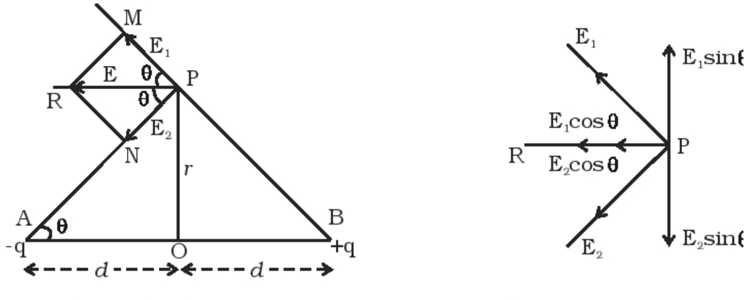
If the point P is far away from the dipole, then d <<r

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4rd}{r^4}\right] = \frac{q}{4\pi\varepsilon_0}\frac{4d}{r^3} \text{ along BP.}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} \text{ along BP.} \approx \frac{2k\vec{p}}{r^3} \qquad [\because \text{ Electric dipole moment p = q x 2d}]$$

$\vec{E}$ acts in the direction of dipole moment.

1.2.6 Electric field due to an electric dipole at a point on the equatorial line.



(a) Electric field at a point on equatorial line *(b) The components of the electric field*

Fig 1.8

Consider an electric dipole AB. Let 2d be the dipole distance and p be the dipole moment. P is a point on the equatorial line at a distance from the midpoint O of the dipole.(fig1.8a)

Electric field at a point P due to the charge +q of the dipole,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2} \text{ along BP.}$$

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+d^2} \text{ along BP. } (\because BP^2 = OP^2 + OB^2)$$

Electric field at a point P due to the charge +q of the dipole,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2} \text{ along BP.}$$

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+d^2} \text{ along BP. } (\because BP^2 = OP^2 + OB^2)$$

Electric field ($\vec{E}_2$) at a point P due to the charge –q of the dipole

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2} \text{ along PA.}$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+d^2} \text{ along PA}$$

The magnitudes of E_1 and E_2 are equal. Resolving E_1 and E_2 into their horizontal and vertical components (fig1.8b), the vertical components $E_1 \sin\theta$ and $E_2 \sin\theta$ are equal and opposite, therefore they cancel each other. The horizontal components $E_1 \cos\theta$ and $E_2 \cos\theta$ will get added along PR. Resultant electric field at the point P due to the dipole is

$$E = E_1\cos\theta + E_2\cos\theta \text{ (along PR)}$$

$$= 2E_1\cos\theta \quad (\because E_1 = E_2)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+d^2} \times 2\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+d^2}\frac{2d}{(r^2+d^2)^{1/2}} \qquad (\because \cos\theta = \frac{d}{\sqrt{r^2+d^2}})$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+d^2)^{\frac{3}{2}}} \qquad (\because p = q.2d)$$

For a dipole, d is very small when compared to r

$$\therefore \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} \text{ (along PR)} \quad \approx -\frac{k\vec{p}}{r^3}$$

The direction of $\vec{E}$ is along PR, parallel to the axis of the dipole and directed opposite to the direction of dipole moment.

1.2.7 Electric dipole in a uniform electric field

Consider a dipole AB of dipole moment p placed at an angle θ in an uniform electric field E (Fig 1.9). The charge +q experience a force qE in the direction of the field. The charge –q experiences an equal force in the opposite direction. Thus the net force on the dipole is zero.

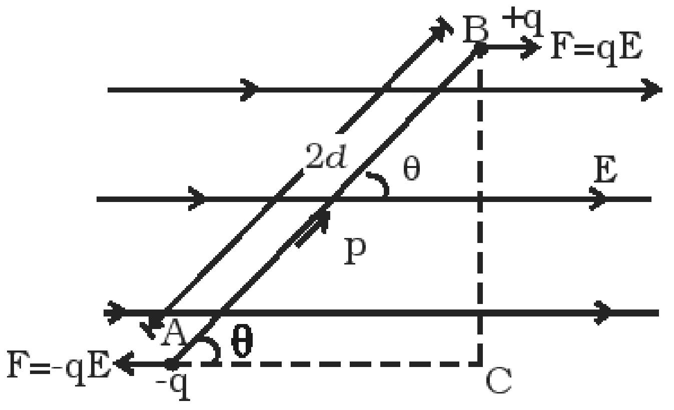


Fig 1.9 Dipole in a uniform field

The two equal and unlike parallel forces are not passing through the same point, resulting in a torque on the dipole, which tends to set the dipole in the direction of the electric field.

The magnitude of torque is,

τ = One of the forces x perpendicular distance between the forces

= F x 2d sin θ

= qE x 2d sin θ

= pE sin θ (∵ q × 2d = P)

In vector notation, $\vec{\tau} = \vec{p} \times \vec{E}$

Note : If the dipole is placed in a non-uniform electric field at an angle θ, in addition to a torque, it also experiences a force.

1.2.8 Electric potential energy of an electric dipole in an electric field.

Electric potential energy of an electric dipole in an electrostatic field is the work done in rotating the dipole to the desired position in the field. When an electric dipole of dipole moment p is at an angle θ with the electric field E, the torque on the dipole is τ = pE sin θ

Work done in rotating the dipole through d θ against the torque acting on it is

$$dw = \tau\, d\theta = pE \sin\theta\, d\theta$$

The total work done in rotating the dipole through an angle θ is

$$W = \int dw$$

$$W = pE \int \sin\theta\, d\theta = -pE\cos\theta$$

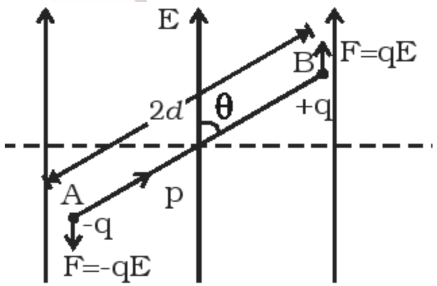


Fig 1.10 Electric potential energy of dipole

This work done is the potential energy (U) of the dipole.

$$\therefore U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

(i) When the dipole is held perpendicular to the external field θ = 90° (position of Zero energy),

$$U = -pE\cos 90° = 0$$

This shows the dipole has a zero potential energy when it is held perpendicular to the external field. If we held the dipole perpendicular to the electric field and bring it from infinity into field, then the work done on charge +q by the external agent is equal to the work done on charge – q. The net work done on the dipole will be zero and hence its potential energy is zero.

(ii) When the dipole is aligned parallel to the external field, θ = 0° (position of stable equilibrium),

$$U = -pE\cos 0° = -pE$$

This shows the dipole has a minimum potential energy when it is aligned with the field.

(iii) When the dipole is antiparallel to the external field, θ = 180° (position of unstable equilibrium),

$$U = -pE\cos 180° = pE$$

This shows the dipole has a maximum potential energy when it is antiparallel to the external field.

A dipole in the electric field experiences a torque ($\vec{\tau} = \vec{p} \times \vec{E}$) which tends to align the dipole in the field direction, dissipating its potential energy in the form of heat to the surroundings.

1.3 Electric potential

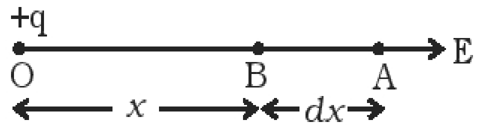


Fig1.11 Electric potential

Let a charge +q be placed at a point O (fig1.11). A and B are two points, in the electric field. When an positive test charge Δq is moved from A to B against the electric force, a work $W_{A\rightarrow B}$ is done. Then potential difference between these two points is

$$\Delta V = V_B - V_A = \frac{W_{A\rightarrow B}}{\Delta q}$$

The potential difference between two points in an electric field is defined as the amount of work done in moving an unit positive charge from one point to the other against the electric force.

The unit of potential difference is volt. The potential difference between two points is 1 volt if 1 joule of work is done in moving 1 Coulomb of charge from one point to another against the electric force.

The electric potential in an electric field at a point is defined as the amount of work done in moving a unit positive charge from infinity to that point against the electric forces.

If the point lie A lie on infinity, then V_A = 0, So, potential at point B is

$$V = V_B = V_B - V_\infty = \frac{W_{\infty\rightarrow B}}{\Delta q}$$

Relation between electric field and potential

Let the small distance between A and B is dx. Work done in moving a unit positive charge from A to B is

$$dV = \vec{E}\cdot\vec{dx}$$

The work has to be done against the force of repulsion in moving a unit positive charge towards the charge +q. Hence,

$$dV = -E\cdot dx$$

$$E = \frac{-dV}{dx}$$

The change of potential with distance is known as potential gradient; hence the electric field is equal to the negative gradient of potential. The negative sign indicates that the potential decreases in the direction of electric field. The unit of electric intensity can also be expressed as Vm^{-1}.

1.3.1 Electric potential at a point due to a point charge

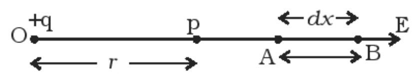


Fig 1.12 Electric potential due to a point charge

Let +q be an isolated point charge situated in air at O. P is a point at a distance r from +q. Consider two points A and B at distances x and x + dx from the point O (Fig1.12).

The potential difference between A and B is, dV = −E dx

$$dV = -\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}\,dx \qquad \left(\because E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}\right)$$

The negative sign indicates that the work is done against the electric force.

The electric potential at the point P due to the charge +q is the total work done in moving a unit positive charge from infinity to that point.

$$V = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}\, dx = \frac{q}{4\pi\varepsilon_0 r}$$

1.3.2 Electric potential at a point due to an electric dipole

Two charges –q at A and +q at B separated by a small distance 2d constitute an electric dipole and its dipole moment is p (Fig 1.13).

Let P be the point at a distance r from the midpoint of the dipole O and θ is the angle between PO and the axis of the dipole OB. Let r_1 and r_2 be the distances of the point P from +q and –q charges respectively.

Potential at P due to charge (+q) = $\frac{1}{4\pi\varepsilon_0}\frac{q}{r_1}$

Potential at P due to charge (–q) = $\frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{r_2}\right)$

Total potential at P due to dipole is, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_2}$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad ..(1)$$

Applying cosine law,

$$r_1^2 = r^2 + d^2 - 2rd\cos\theta$$

$$r_1^2 = r^2\left(1 - 2d\frac{\cos\theta}{r} + \frac{d^2}{r^2}\right)$$

Since d is very much smaller than r, $\frac{d^2}{r^2}$ can be neglected

$$r_1 = r\left(1 - 2d\frac{\cos\theta}{r}\right)^{1/2}$$

$$\text{or, } \frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2d}{r}\cos\theta\right)^{-1/2}$$

Using the Binomial theorem and neglecting higher powers,

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{d}{r}\cos\theta\right) \quad ....(2)$$

Similarly,

$$r_2^2 = r^2 + d^2 - 2rd\cos(180° - \theta)$$

$$\text{or} \quad r_2^2 = r^2\left(1 + 2d\frac{\cos\theta}{r} + \frac{d^2}{r^2}\right)$$

$$r_2 = r\left(1 + 2d\frac{\cos\theta}{r}\right)^{1/2} \qquad (\because \frac{d^2}{r^2} \text{ is negligible})$$

$$\text{or, } \frac{1}{r_2} = \frac{1}{r}\left(1 + \frac{2d}{r}\cos\theta\right)^{-1/2}$$

Using the Binomial theorem and neglecting higher powers,

$$\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{d}{r}\cos\theta\right) \quad ....(3)$$

Substituting equation (2) and (3) in equation (1) and simplifying

$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left(1 + \frac{d}{r}\cos\theta - 1 + \frac{d}{r}\cos\theta\right)$$

$$V = \frac{q\,2d\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad ....(4) \qquad \Rightarrow V \approx \frac{k\vec{p}.\vec{r}}{r^3}$$

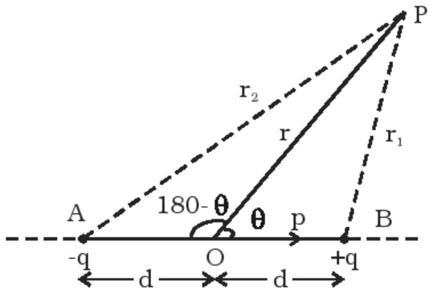


Fig 1.13 Potential due to a dipole

Special cases:

1. When the point P lies on the axial line of the dipole on the side of +q, then θ = 0°

$\therefore \quad V = \frac{p}{4\pi\varepsilon_0 r^2}$

2. When the point P lies on the axial line of the dipole on the side of –q, then θ = 180°

$\therefore \quad V = -\frac{p}{4\pi\varepsilon_0 r^2}$

3. When the point P lies on the equatorial line of the dipole, then, θ = 90°, $\therefore$ V = 0

1.3.3 Electric potential energy

The electric potential energy of two point charges is equal to the work done to assemble the charges or work done in bringing each charge or work done in bringing a charge from infinite distance.

Let us consider a point charge q_1, placed at A (Fig1.14a). The potential at a point B at a distance r from the charge q_1 is

$$V = \frac{q_1}{4\pi\varepsilon_0 r}$$

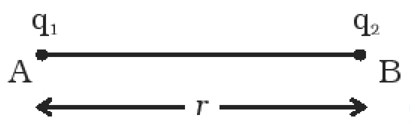


Fig 1.14a Electric potential energy

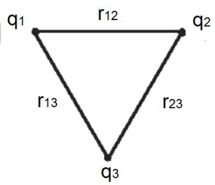


Fig 1.14b Potential energy of system of charges

Another point charge q_2 is brought from infinity to the point B. Now the work done on the charge q_2 is stored as electrostatic potential energy (U) in the system of charges q_1 and q_2 .

$\therefore$ work done, W = Vq_2

Potential energy (U) = $\frac{q_1 q_2}{4\pi\varepsilon_0 r}$

Keeping q_2 at B, if the charge q_1 is imagined to be brought from infinity to the point A, the same amount of work is done.

Also, if both the charges q_1 and q_2 are brought from infinity, to points A and B respectively, separated by a distance r, then potential energy of the system is the same as the previous cases.

For a system containing more than two charges (Fig1.14b), the potential energy (U) is given by

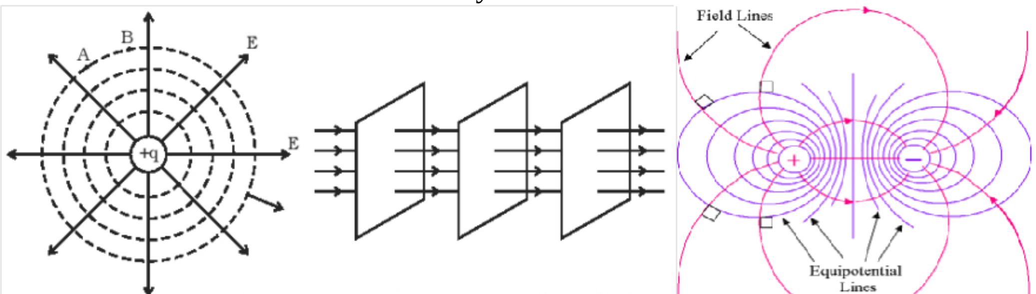
$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

1.3.4 Equipotential Surface

If all the points of a surface are at the same electric potential, then the surface is called an equipotential surface.

(i) In case of an isolated point charge, all points equidistant from the charge are at same potential. Thus, equipotential surfaces in this case will be a series of concentric spheres with the point charge as their centre (Fig. 1.15a). The potential, will however be different for different spheres. If the charge is to be moved between any two points on an equipotential surface through any path, the work done is zero. This is because the potential difference between two points A and B is defined as $V_B - V_A$ =If $V_A = V_B$ then W_{AB} = 0. Hence the electric field lines must be normal to an equipotential surface.

(ii) In case of uniform field, equipotential surfaces are the parallel planes with their surfaces perpendicular to the lines of force as shown in fig1.15b.



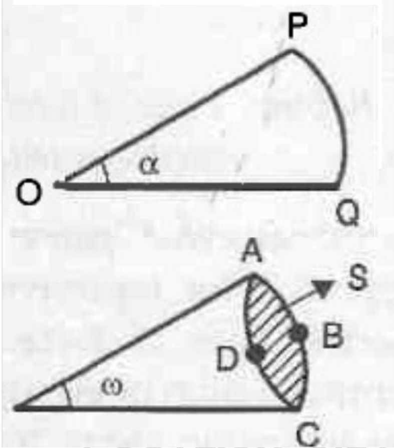
(a) Equipotential surface (spherical)

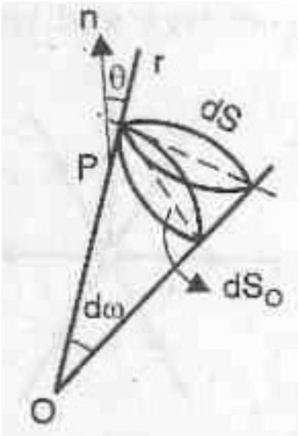
(b) For a uniform field (plane)

Fig 1.15

1.4 Gauss's law and its applications

Solid angle:





A plane consists of a number of lines. We consider a curve PQO (shown in figure). This curve makes an angle α at point O.

A solid consists of a number of planes. We consider a plane ABCD. If all points A,B,C,D and the periphery of the plane are joined with a point O. It is called solid angle.

Then this plane makes solid angle 'ω' at point O. Its unit is steradian.

Solid angle at a cone is defined as area intercepted by the cone on a sphere of unit radius having its centre at the vertex of the cone. The solid angle subtended by an area S at O is

$$d\omega = \int_S \frac{ds\cos\theta}{r^2}$$

Solid angle subtended by a closed surface at an internal point is 4π.

Electric flux

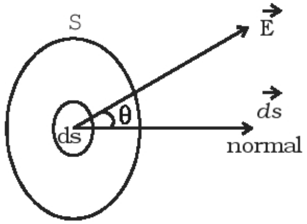


Fig1.16 Electric flux

Consider a closed surface S in a non-uniform electric field (Fig1.16). Consider a very small area ds on this surface. The direction of ds is drawn normal to the surface outward.

The electric field over ds is supposed to be a constant E. E and ds make an angle θ with each other.

The electric flux is defined as the total number of electric lines of force, crossing through the given area. The electric flux dϕ through the area ds is,

$$d\phi = \vec{E}.\vec{ds} = E\,ds\cos\theta$$

The total flux through the closed surface S is obtained by integrating the above equation over the surface

$$\phi = \oint d\phi = \oint \vec{E}.\vec{ds}$$

The circle on the integral indicates that, the integration is to be taken over the closed surface. The electric flux is a scalar quantity. Its unit is N m^2 C^{-1}

1.4.1 Gauss's law

The law relates the flux through any closed surface and the net charge enclosed within the surface. The law states that the total flux of the electric field E over any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface. $\phi = \frac{q}{\varepsilon_0}$

This closed imaginary surface is called Gaussian surface. Gauss's law tells us that the flux of E through a closed surface S depends only on the value of net charge inside the surface and not on the location of the charges. Charges outside the surface will not contribute to flux.

1.4.2 Electric Field due to an infinite long straight charged wire

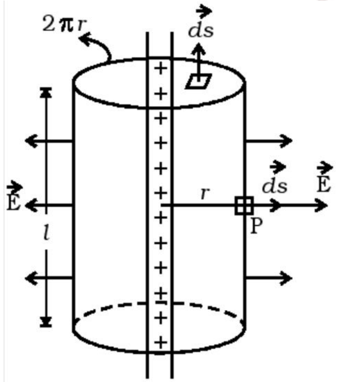


Fig 1.17 Infinitely long straight charged wire

Consider an uniformly charged wire of infinite length having a constant linear charge density λ (charge per unit length). Let P be a point at a distance r from the wire (Fig1.17) and E be the electric field at the point P. A cylinder of length l, radius r, closed at each end by plane caps normal to the axis is chosen as Gaussian surface. Consider a very small area ds on the Gaussian surface. By symmetry, the magnitude of the electric field will be the same at all points on the curved surface of the cylinder and directed radially outward. E and ds are along the same direction.

The electric flux (ϕ) through curved surface = $\oint E\,ds\cos\theta$

$\phi = \oint \vec{E}.\vec{ds} = \oint E ds$ [$\because \theta = 0; \cos\theta = 1$]

$= E(2\pi rl)$ ($\because$ the surface area of the curved part is $2\pi rl$)

Since E and ds are right angles to each other, the electric flux through the plane caps = 0

$\therefore$ Total flux through the Gaussian surface, $\phi = E.(2\pi rl)$

The net charge enclosed by Gaussian surface is, $q = \lambda l$

$\therefore$ By Gauss's law, $E(2\pi rl) = \frac{\lambda l}{\varepsilon_0}$ or $E = \frac{\lambda}{2\pi\varepsilon_0 r} \approx \frac{2k\lambda}{r}$

The direction of electric field E is radially outward, if line charge is positive and inward, if the line charge is negative.

1.4.3 Electric field due to an infinite charged plane sheet

Consider an infinite plane sheet of charge with surface charge density σ . Let P be a point at a distance r from the sheet (fig1.18) and E be the electric field at P. Consider a Gaussian surface in the form of cylinder of cross sectional area A and length 2r perpendicular to the sheet of charge.

By symmetry, the electric field is at right angles to the end caps and away from the plane. Its magnitude is the same at P and at the other cap at P′ .

Therefore, the total flux through the closed surface is given by

$$\phi = \left[\oint \vec{E}.\vec{ds}\right]_P + \left[\oint \vec{E}.\vec{ds}\right]_{P'} \quad (\because \theta = 0, \cos\theta = 1)$$

$$\therefore \phi = EA + EA = 2EA$$

If σ is the charge per unit area in the plane sheet, then the net positive charge q within the Gaussian surface is,

$$q = \sigma A$$

Using Gauss's law,

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$\therefore E = \frac{\sigma}{2\varepsilon_0}$$

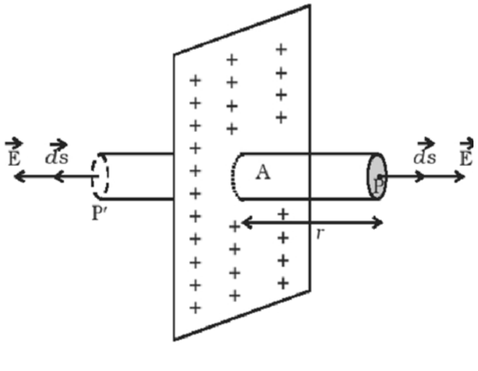


Fig 1.18 Infinite plane sheet

1.4.4 Electric field due to two parallel charged sheets

Consider two plane parallel infinite sheets with equal and opposite charge densities $+\sigma$ and $-\sigma$ shown in fig1.19(a). The magnitude of electric field on either side of a plane sheet of charge is $E = \sigma/(2\varepsilon_0)$ and acts perpendicular to the sheet, directed outward (if the charge is positive) or inward (if the charge is negative).

(i) When the point P_1 is in between the sheets, the field due to two sheets will be equal in magnitude and in the same direction. The resultant field at P_1 is,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} \text{ (towards the right)}$$

(ii) At a point P_2 outside the sheets, the electric field will be equal in magnitude and opposite in direction. The resultant field at P_2 is,

$$\vec{E} = \vec{E}_1 - \vec{E}_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0.$$

(iii) Fig 1.19 (b) shows electric lines of force when a metallic plate is introduced between plate of a charged capacitor. Fig 1.19 (c) shows electric lines of force when a dielectric plate is introduced between plates of a charged capacitor then, number of lines of forces in dielectric is lesser than that in case of vacuum space

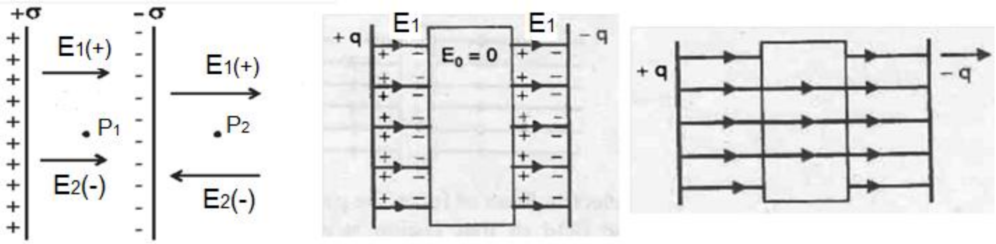


Fig 1.19 (a) Field due to two parallel sheets 1.19(b) 1.19(c)

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1.4.5 Electric field due to uniformly charged spherical shell

Case (i) At a point outside the shell.

Consider a charged shell of radius R(Fig1.20a). Let P be a point outside the shell, at a distance r from the centre O. Let us construct a Gaussian surface with r as radius. The electric field E is normal to the surface.

The flux crossing the Gaussian sphere normally in an outward direction is,

$$\phi = \int_s \vec{E}.\vec{ds}$$

since angle between E and ds is zero

$$\phi = \int_s E\, ds$$

$$= E\,(4\pi r^2)$$

By Gauss's law, $E.(4\pi r^2) = \frac{q}{\varepsilon_0}$

or $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \approx \frac{kq}{r^2}$

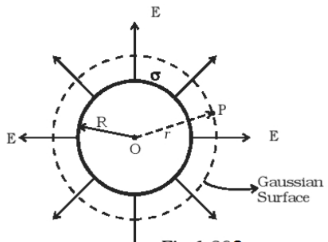


Fig 1.20a

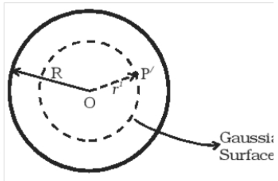


Fig 1.20b Field at a point inside the shell

It can be seen from the equation that, the electric field at a point outside the shell will be the same as if the total charge on the shell is concentrated at its centre.

Case (ii) At a point on the surface.

The electric field E for the points on the surface of charged spherical shell is,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} \quad (\because r = R)$$

Case (iii) At a point inside the shell.

Consider a point P' inside the shell at a distance r' from the centre of the shell. Let us construct a Gaussian surface with radius r'. The total flux crossing the Gaussian sphere normally in an outward direction is

$$\phi = \int_s \vec{E}.\vec{ds} = \int_s E\, ds = E\,(4\pi r'^2)$$

since there is no charge enclosed by the gaussian surface, according to Gauss's Law

$$E.(4\pi r^2) = \frac{q}{\varepsilon_0} = 0 \qquad \therefore E = 0$$

(i.e) the field due to a uniformly charged thin shell is zero at all points inside the shell.

1.4.6 Electrostatic shielding

It is the process of isolating a certain region of space from external field. It is based on the fact that electric field inside a conductor is zero.

During a thunder accompanied by lightning, it is safer to sit inside a bus than in open ground or under a tree. The metal body of the bus provides electrostatic shielding, where the electric field is zero. During lightning the electric discharge passes through the body of the bus.

1.5 Electrostatic induction

It is possible to obtain charges without any contact with another charge. They are known as induced charges and the phenomenon of producing induced charges is known as electrostatic induction. It is used in electrostatic machines like capacitors. Fig 1.21 shows the steps involved in charging a metal sphere by induction.

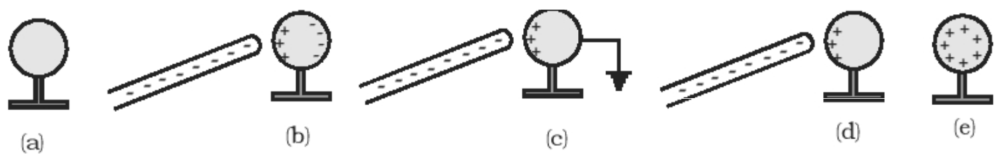


Fig 1.21 Electrostatic Induction

(a) There is an uncharged metallic sphere on an insulating stand.

(b) When a negatively charged plastic rod is brought close to the sphere, the free electrons move away due to repulsion and start pilling up at the farther end. The near end becomes positively charged due to deficit of electrons. This process of charge distribution stops when the net force on the free electron inside the metal is zero (this process happens very fast).

(c) When the sphere is grounded, the negative charge flows to the ground. The positive charge at the near end remains held due to attractive forces.

(d) When the sphere is removed from the ground, the positive charge continues to be held at the near end.

(e) When the plastic rod is removed, the positive charge spreads uniformly over the sphere.

1.5.1 Capacitance of a conductor

When a charge q is given to an isolated conductor, its potential will change. The change in potential depends on the size and shape of the conductor. The potential of a conductor changes by V, due to the charge q given to the conductor. $q \propto V$ or $q = CV$

i.e. $C = q/V$

Here C is called as capacitance of the conductor. The capacitance of a conductor is defined as the ratio of the charge given to the conductor to the potential developed in the conductor.

The unit of capacitance is farad. A conductor has a capacitance of one farad, if a charge of 1 coulomb given to it, rises its potential by1 volt.

The practical units of capacitance are μ F and pF.

Principle of a capacitor

Consider an insulated conductor (Plate A) with a positive charge 'q' having potential V (fig1.22a). The capacitance of A is C = q/V. When another insulated metal plate B is brought near A, negative charges are induced on the side of B near A. An equal amount of positive charge is induced on the other side of B (fig1.22b). The negative charge in B decreases the potential of A. The positive charge in B increases the potential of A. But the negative charge on B is nearer to A than the positive charge on B. So the net effect is that, the potential of A decreases. Thus the capacitance of A is increased.

If the plate B is earthed, positive charges get neutralized (fig1.22c). Then the potential of A decreases further. Thus the capacitance of A is considerably increased.

The capacitance depends on the geometry of the conductors and nature of the medium. A capacitor is a device for storing electric charges.

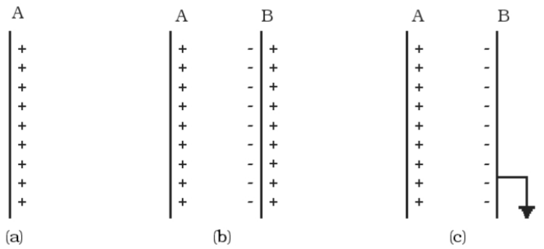


Fig 1.22 Principle of capacitor

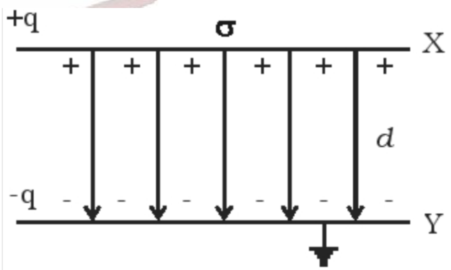


Fig 1.23 Parallel plate capacitor

1.5.2 Capacitance of a parallel plate capacitor

The parallel plate capacitor consists of two parallel metal plates X and Y each of area A, separated by a distance d, having a surface charge density σ (fig1.23). The medium between the plates is air. A charge +q is given to the plate X. It induces a charge –q on the upper surface of earthed plate Y. When the plates are very close to each other, the field is confined to the region between them. The electric lines of force starting from plate X and ending at the plate Y are parallel to each other and perpendicular to the plates.

By the application of Gauss's law, electric field at a point between the two plates is,

$$E = \frac{\sigma}{\varepsilon_0}$$

Potential difference between the plates X and Y is

$$V = \int_d^0 -E dr = \int_d^0 -\frac{\sigma}{\varepsilon_0} dr = \frac{\sigma d}{\varepsilon_0}$$

The capacitance (C) of the parallel plate capacitor

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma d/\varepsilon_0} \qquad [\text{since, } \sigma = \frac{Q}{A}]$$

$$\therefore C = \frac{\varepsilon_0 A}{d}$$

The capacitance is directly proportional to the area (A) of the plates and inversely proportional to their distance of separation (d).

1.5.3 Dielectrics and polarisation

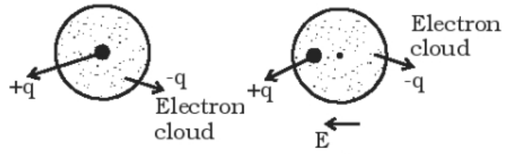
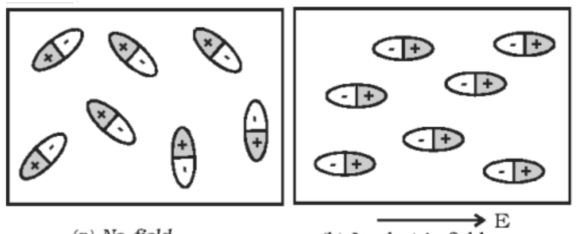


Fig 1.24 Induced dipole



(a) No field *(b) In electric field*

Fig1.25 Polar molecules

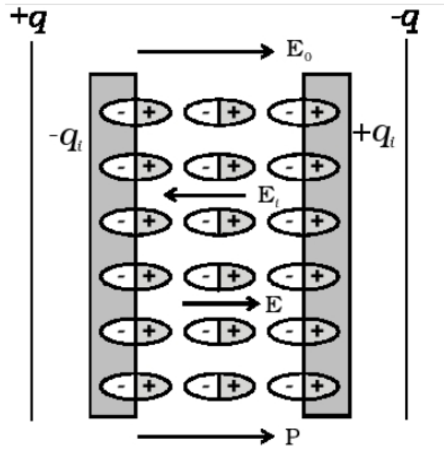


Fig1.26 Polarisation of dielectric material

Dielectrics

A dielectric is an insulating material in which all the electrons are tightly bound to the nucleus of the atom. There are no free electrons to carry current. Ebonite, mica and oil are few examples of dielectrics. The electrons are not free to move under the influence of an external field.

Polarisation

A nonpolar molecule is one in which the centre of gravity of the positive charges (protons) coincide with the centre of gravity of the negative charges (electrons). Example: O_2, N_2, H_2. The nonpolar molecules do not have a permanent dipole moment.

If a non polar dielectric is placed in an electric field, the centre of charges get displaced. The molecules are then said to be polarised and are called induced dipoles. They acquire induced dipole moment p in the direction of electric field (fig1.24).

A polar molecule is one in which the centre of gravity of the positive charges is separated from the centre of gravity of the negative charges by a finite distance. Examples : N_2O, H_2O, HCl, NH_3. They have a permanent dipole moment. In the absence of an external field, the dipole moments of polar molecules orient themselves in random directions. Hence no net dipole moment is observed in the dielectric. When an electric field is applied, the dipoles orient themselves in the direction of electric field. Hence a net dipole moment is produced (fig1.25).

The alignment of the dipole moments of the permanent or induced dipoles in the direction of applied electric field is called polarisation or electric polarisation.

The magnitude of the induced dipole moment p is directly proportional to the external electric field E.

∴ p α E or p = α E, where α is the constant of proportionality and is called molecular polarisability.

The maximum external electric field the dielectric can withstand without dielectric beakdown is called dielectric strength. SI unit Vm^{-1}.

1.5.4 Polarisation of dielectric material

Consider a parallel plate capacitor with +q and –q charges. Let E_0 be the electric field between the plates in air. If a dielectric slab is introduced in the space between them, the dielectric slab gets polarised. Suppose $+q_i$ and $-q_i$ be the induced surface charges on the face of dielectric opposite to the plates of capacitor (fig1.26). These induced charges produce their own field E_i which opposes the electric field E_o. So, the resultant field, E < E_o. But the direction of E is in the direction of E_o.

∴ $E = E_o + (-E_i)$ (∵ E_i is opposite to the direction of E_o)

1.5.5 Capacitance of a parallel plate capacitor with a dielectric medium.

Consider a parallel plate capacitor having two conducting plates X and Y each of area A, separated by a distance d apart. X is given a positive charge so that the surface charge density on it is σ and Y is earthed.

Let a dielectric slab of thick-ness t and relative permittivity ε_r be introduced between the plates (fig1.27).

Thickness of dielectric slab = t

Thickness of air gap = (d - t)

Electric field at any point in the air between the plates,

$$E = \frac{\sigma}{\varepsilon_0}$$

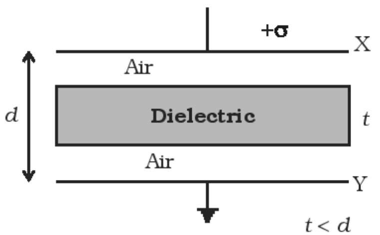


Fig 1.27 Dielectric in capacitor

Electric field at any point, in the dielectric slab $E' = \frac{\sigma}{\varepsilon_r\varepsilon_0}$

The total potential difference between the plates, is the work done in crossing unit positive charge from one plate to another in the field E over a distance (d–t) and in the field E' over a distance t, then

$$V = E(d-t) + E't = \frac{\sigma}{\varepsilon_0}(d\text{-}t) + \frac{\sigma t}{\varepsilon_r\varepsilon_0} = \frac{\sigma}{\varepsilon_0}\left[(d-t) + \frac{t}{\varepsilon_r}\right]$$

The charge on the plate X, q = σ A

Hence the capacitance of the capacitor is,

$$C = \frac{q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}\left[(d-t)+\frac{t}{\varepsilon_r}\right]} = \frac{\varepsilon_0 A}{(d-t)+\frac{t}{\varepsilon_r}}$$

Effect of dielectric

In capacitors, the region between the two plates is filled with dielectric like mica or oil. The capacitance of the air filled capacitor, $C = \frac{\varepsilon_0 A}{d}$

The capacitance of the dielectric filled capacitor, $C' = \frac{\varepsilon_r\varepsilon_0 A}{d}$

$\frac{C'}{C} = \varepsilon_r$ or, $C' = \varepsilon_r C$

since, $\varepsilon_r > 1$ for any dielectric medium other than air, the capacitance increases, when dielectric is placed.

Capacitance of a parallel plate capacitor with a conducting medium

Consider a conducting slab of thickness't' is introduced without touching between the plates of a parallel plate capacitor separated by a distance d (t<d).

For a parallel plate capacitor when air/vacuum is in between the plates $C_0 = \frac{\varepsilon_0 A}{d}$. Since electric field inside a conducting stab is zero, hence electric field exist only between the space (d-t)

∴ $V = E_0 (d - t)$ Where E_0 is the electric field exist between the plates

And $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$ Where A is the area of each plate

∴ $V = \frac{q}{A\varepsilon_0}(d - t)$

Hence capacitor of a parallel plate capacitor $C = \frac{q}{V} = \frac{A\varepsilon_0}{d-t} = \frac{C_0}{(1-\frac{t}{d})}$

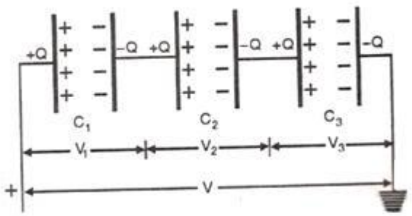
1.5.6 Capacitors in series and parallel

(i) Capacitors in series

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in series (fig1.28). Let V be the potential difference applied across the series combination. Each capacitor carries the same amount of charge q. Let V_1, V_2, V_3 be the potential difference across the capacitors C_1, C_2, C_3 respectively. Thus $V = V_1 + V_2 + V_3$

The potential difference across each capacitor is,

$V_1 = \frac{q}{c_1}$; $V_2 = \frac{q}{c_2}$; $V_3 = \frac{q}{c_3}$;



∴ $V = \frac{q}{c_1} + \frac{q}{c_2} + \frac{q}{c_3} = q\left[\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}\right]$

If C_S be the effective capacitance of the series combination, it should acquire a charge q when a voltage V is applied across it.

i.e. $V = \frac{q}{Cs}$

∴ $q\left[\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}\right] = \frac{q}{Cs}$

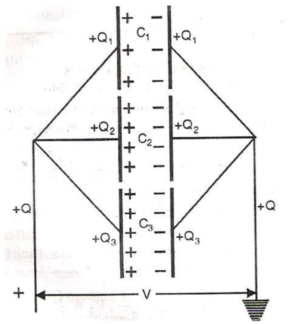
∴ $\frac{1}{Cs} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$

when a number of capacitors are connected in series, the reciprocal of the effective capacitance is equal to the sum of reciprocal of the capacitance of the individual capacitors.

(ii) Capacitors in parallel

Consider three capacitors of capacitances C_1, C_2 and C_3 connected in parallel (fig1.29). Let this parallel combination be connected to a potential difference V. The potential difference across each capacitor is the same. The charges on the three capacitors are,

$q_1 = C_1V$, $q_2 = C_2V$, $q_3 = C_3V$.



The total charge in the system of capacitors is

$q = q_1 + q_2 + q_3$

∴ $q = C_1V + C_2V + C_3V$

But $q = C_p.V$ where C_p is the effective capacitance of the system

∴ $C_pV = V(C_1 + C_2 + C_3)$

∴ $C_P = C_1 + C_2 + C_3$

Hence the effective capacitance of the capacitors connected in parallel is the sum of the capacitances of the individual capacitors.

1.5.7 Energy stored in a capacitor

The capacitor is a charge storage device. Work has to be done to store the charges in a capacitor. This work done is stored as electrostatic potential energy in the capacitor.

Let q be the charge and V be the potential difference between the plates of the capacitor. If dq is the additional charge given to the plate, then work done is,

$$dw = Vdq$$

$$dw = \frac{q}{c}\,dq \qquad (\because V = \frac{q}{c_1})$$

Total work done to charge a capacitor is

$$W = \int dW = \int \frac{q}{c}\,dq = \frac{1}{2}\frac{q^2}{C}$$

This work done is stored as electrostatic potential energy (U) in the capacitor.

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 \quad (\because q = CV)$$

This energy is recovered if the capacitor is allowed to discharge.

On connecting two charged capacitors:

When two conductors are connected the charges flow from higher potential plate to lower potential plate till they reach a common potential.

Common Potential: A capacitor of capacitance C_1 and potential V_1 is connected to another capacitor of capacitance C_2 and potential V_2. The charge flow from higher potential to lower potential and it reach an in between value V such that

$$V = \frac{Total\ charge}{Totl\ capacitance} \qquad \text{or } V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Loss of Energy on connecting two conductors: A capacitor of capacitance C_1 and potential V_1 is connected to another capacitor of capacitance C_2 and potential V_2. The charge flow from higher potential to lower potential and in this process it looses some energy as charge has to do some work while passing through connecting wire. The energy is lost in form of heat of connecting wire.

Expression for energy lost : In the above two capacitors the energy contained in the two before connection,

$$E_1 = \frac{1}{2}C_1(V_1)^2 + \frac{1}{2}C_2(V_2)^2 \quad \ldots\ldots\ (i)$$

Common Potential after connection, $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

Combined capacitance $C_1 + C_2$

Energy in combination : $E_2 = \frac{1}{2}(C_1 + C_2)\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2$

Hence Loss in energy : $E_1 - E_2 = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

It is a positive number which confirm that there is loss of energy in transfer of charges.

Common Potential: $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ Loss of energy: $\Delta E = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

1.5.8 Applications of capacitors.

a) They are used in the ignition system of automobile engines to eliminate sparking.
b) They are used to reduce voltage fluctuations in power supplies and to increase the efficiency of power transmission.
c) Capacitors are used to generate electromagnetic oscillations and in tuning the radio circuits.

1.5.9 Distribution of charges on a conductor and action of points

Let us consider two conducting spheres A and B of radii r_1 and r_2 respectively connected to each other by a conducting wire (fig1.30). Let r_1 be greater than r_2 . A charge given to the system is distributed as q_1 and q_2 on the surface of the spheres A and B. Let σ_1, σ_2 be the charge densities on the sphere A and B.

The potential at A,

$$V_1 = \frac{q_1}{4\pi\varepsilon_0 r_1}$$

The potential at B,

$$V_2 = \frac{q_2}{4\pi\varepsilon_0 r_2}$$

Since they are connected, their potentials are equal

$$\frac{q_1}{4\pi\varepsilon_0 r_1} = \frac{q_2}{4\pi\varepsilon_0 r_2}$$

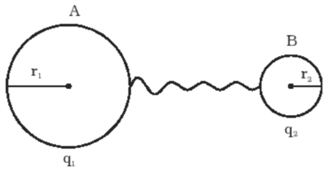


Fig 1.30 Distribution of charges

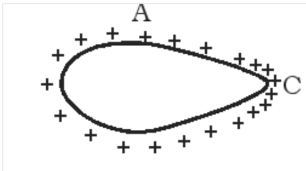


Fig 1.31 Action of point

$$\sigma_1 r_1 = \sigma_2 r_2 \qquad (\because q_1 = 4\pi r_1^2\sigma_1 \text{ and } q_2 = 4\pi r_2^2\sigma_2)$$

i.e., σ r is a constant. From the above equation it is seen that, smaller the radius, larger is the charge density.

In case of conductor, shaped as in (fig 1.31). the distribution is not uniform. The charges accumulate to a maximum at the pointed end where the curvature is maximum or the radius is minimum. It is found experimentally that a charged conductor with sharp points on its surface, loses its charge rapidly.

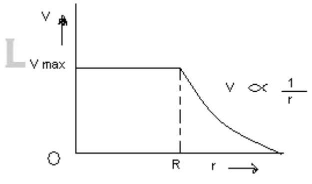
The reason is that the air molecules which come in contact with the sharp points become ionized. The positive ions are repelled and the negative ions are attracted by the sharp points and the charge in them is therefore reduced.

Thus, the leakage of electric charges from the sharp points on the charged conductor is known as action of points or corona discharge. This principle is made use of in the electrostatic machines for collecting charges and in lightning arresters (conductors).

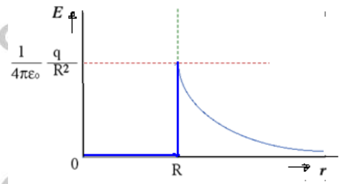
1.6 Lightning conductor

This is a simple device used to protect tall buildings from the lightning. It consists of a long thick copper rod passing through the building to ground. The lower end of the rod is connected to a copper plate buried deeply into the ground. A metal plate with number of spikes is connected to the top end of the copper rod and kept at the top of the building.

When a negatively charged cloud passes over the building, positive charge will be induced on the pointed conductor. The positively charged sharp points will ionize the air in the vicinity. This will partly neutralize the negative charge of the cloud, thereby lowering the potential of the cloud. The negative charges that are attracted to the conductor travels down to the earth. Thereby preventing the lightning stroke from the damage of the building.



Relation of V & R for spherical shell



Relation of E & R for spherical shell